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RESEARCH MEMORANDUM

ANALYSIS OF COOLANT FLOW AND PRESSURE REQUIREMENTS FOR

A RETURN-FLOW TURBINE ROTOR BLADE DESIGN USING

HYDROGEN, HELIUM, OR AIR AS COOLANT

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RETURN-FLOW TURBINE ROTOR BLADE DESIGN USING

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SUMMARY

An analysis was made to determine the coolant flow and pressure requirements of a return-flow turbine rotor blade which utilizes hydrogen, helium, or air as the coolant. The return-flow blade design selected for this study consists of a hollow blade shell capped at the blade tip and a hollow insert. The coolant flows radially upward between the blade shell and insert and then radially downward through the hollow insert to be discharged at the blade root. Required coolant flow and pressure changes through the blade were obtained for conditions representative of a high-altitude supersonic turbojet engine (flight Mach number of 2.5 and altitudes of 50,000 and 80,000 ft) with a turbine-inlet temperature of 3000° R and a turbine blade root stress of 50,000 psi.

From this analysis it appears that the return-flow blade design considered would cool turbine rotor blades adequately over a wide range of operating conditions using hydrogen, helium, or air as the coolant. Using air as the coolant requires an increase in the required coolant flow of about 15 and 6 times that required when using hydrogen or helium, respectively. On the basis of preliminary calculations, the use of air as the coolant for the return-flow blade would require the smallest diameter coolant ducting to transport the coolant to and from the turbine blades. Hydrogen and helium require pipes about 1.6 and 1.9 times the diameter of the air ducting. The use of equations which neglect the heat being transferred across the blade insert will result in blade shell temperatures from about 1 to 5 percent lower than those obtained when the heat transfer across the insert is considered.

INTRODUCTION

Up to the present time most of the analytical and experimental investigations concerning the cooling of turbine blades of gas-turbine

engines to enable operation at high gas temperatures and high centrifugal blade stresses have employed either expendable air or recirculating liquids as the coolants (refs. 1 and 2). Both air and liquid coolants and their corresponding coolant systems have their place in the field of aircraft gas-turbine engine application. Another type of coolant system that offers interesting possibilities employs a recirculating gas as the coolant. In this respect, both hydrogen and helium are suitable for such a coolant system because of their excellent heat-transfer characteristics.

This report presents the results of an analysis to determine the coolant flow and pressure requirements for a particular return-flow turbine rotor blade design utilizing hydrogen, helium, or air as the coolant. The return-flow blade design selected for this study consists of a hollow blade shell capped at the blade tip and a hollow insert. The coolant flows radially upward between the blade shell and the insert and then radially downward through the hollow insert to be discharged at the blade root.

The advantages and disadvantages of using an air-cooling system wherein the air is discharged at the blade tip are discussed in detail in reference 3 with relation to the effects of air-cooling on engine specific thrust and thrust specific fuel consumption. Some of the gains and problems that are apparent in recirculating-liquid-cooled systems with special emphasis on engine performance are discussed in reference 4. The use of a recirculating gas in a coolant system offers several attractive advantages. The incremental losses in engine specific thrust or power and the possibility of higher specific fuel consumption that accompany an air-cooling system wherein the air is discharged at the blade tip may be lessened with the recirculating-gas system since this type of cooling system approaches the systems discussed in reference 4 for liquid-cooling. If hydrogen or helium is used as the coolant in the recirculating-gas cooling system, the necessity of large amounts of coolant surface necessary for the air-cooled turbine blades discharging the air at the blade tip may be reduced. Thus, it is possible that a less complicated blade design may result for the recirculating-gas cooling system as compared with the blade designs required for air-cooling systems discharging air at the blade tip. Also, the use of gases in the recirculating cooling system eliminates the problem of attaining extremely high fluid pressures, which are required in a water-cooled blade to avoid overcooling, and it may avoid the difficult sealing problems associated with a liquid-cooling system employing liquid metals, metal salts, or metal hydroxides (ref. 4).

Methods for calculating cooled-blade temperature distributions and, thus, coolant flow requirements are given in references 5 and 6 for air-cooled turbine blades with the air being discharged at the blade tip and in reference 7 for return-flow liquid-cooled blades. The equations of

reference 7 are inadequate for the gaseous-cooled return-flow blade design of this analysis because they are derived for the assumption that the liquid coolant temperature is constant at the average temperature of the coolant. This assumption is valid for a liquid coolant (see ref. 7), but a large coolant temperature change may be obtained for a gaseous coolant flowing in a return-flow blade design. For the return-flow blade design being considered, it is important to know whether or not the heat being transferred across the blade insert is negligible. Thus, it is not certain whether or not the equations of reference 5 which neglect heat transfer across the inserts of air-cooled blades can be adapted to the present analysis. Consequently, an iterative method using available theoretical relations for heat transfer (accounting for heat transfer across the blade insert) and pressure drop was evolved. In this way, the coolant flow and pressure requirements for the return-flow blade design considered may be determined.

This report presents (1) the temperature distribution equations necessary for obtaining the coolant flow requirements of the return-flow turbine rotor blade design being analyzed, (2) the resulting coolant flows and pressure requirements for the return-flow blade using hydrogen, helium, or air as the coolant and being subjected to a range of cooling conditions, and (3) an indication as to whether or not the heat being transferred across the blade insert may be neglected. In addition, the relative size or weight of the ducting required to transport hydrogen, helium, or air to the return-flow blade is indicated.

The analysis was made for conditions representative of a high-altitude supersonic turbojet engine (flight Mach number of 2.5 and altitudes of 50,000 and 80,000 ft) with a turbine-inlet temperature of 3000° R and a turbine blade root centrifugal stress of 50,000 psi in order to impose severe cooling conditions on the return-flow blade. Because of the attractive possibility of using a refrigerated light hydrocarbon fuel for the heat sink (ref. 8), the investigation was conducted over an extremely large range of coolant inlet temperatures from 250° to 1000° R, and the coolant inlet static pressures were varied from 3000 to 9000 pounds per square foot. Because of the large number of variables involved and because the calculation procedure required lengthy iterations, the computations were made on an IBM 650 magnetic drum calculator.

BLADE DESCRIPTION

A brief discussion of air-cooled turbine blade designs from the simple hollow blades used by the Germans in 1945 to the more advanced designs devised by the British and the United States are given in reference 1. As pointed out in both references 1 and 9, it is necessary that the internal surface area be large so that a small quantity of

cooling air may be used to achieve an efficient degree of blade cooling
within pressure drop limitations. Thus, for air-cooling systems wherein
the air is discharged at the blade tip, the simple hollow blade or the
hollow blade with insert offers very inefficient cooling. However,
since the heat-transfer characteristics of hydrogen and helium are ex-
cellent, it was decided that for initial calculations the simple hollow
blade with insert could be used to determine coolant flow requirements
and establish trends for the recirculating type of blade necessary for
hydrogen or helium as the coolant.

Sketches of the return-flow blade considered herein are shown in
figures 1 and 2. The blade consists of a hollow shell with a cap at the
blade tip and an inner shell or insert. The coolant flows radially up-
ward between the blade shell and the insert, downward through the flow
passage formed by the insert, and is discharged at the blade root.

For the present analysis, the spacing between the blade shell and
insert was assumed to be 0.050 inch for the majority of the calculations.
For comparison purposes, spacings of 0.070 and 0.020 inch were also con-
sidered. The axial chord lengths of the blade shell and insert were as-
sumed to be 2.5 and 1.2 inches, respectively. In addition, it was as-
sumed that the ratio of perimeter over chord was constant at 2.34 for
the blade shell and 2.1 for the blade insert; the thickest portion of
the blade profile from the pressure to the suction surface was 0.3 inch;
the flow areas were constant in a spanwise direction; the thickness of
the insert was constant at 0.020 inch; and the blade shell had a metal
area taper ratio of 0.50 from the blade tip to the root where the blade
shell thickness at the tip was 0.020 inch. The blade length was 0.50
foot. The following table gives the values of blade geometry used:

Spacing between blade shell and insert, in.	Mean spacing in passage formed by insert, in.	Flow area between shell and insert, sq ft	Flow area in passage formed by insert, sq ft	Hydraulic diameter between blade shell and insert, ft	Hydraulic diameter in passage formed by insert, ft
0.050	0.090	0.00204	0.00157	0.00833	0.0150
.070	.050	.00285	.00087	.01167	.00833
.020	.100	.00082	.00291	.00333	.0166

ANALYSIS

As pointed out previously, it is important to know whether or not
the heat being transferred across the blade insert of the return-flow

rotor blade design considered herein is negligible. Therefore, heat-transfer equations were derived which account for heat transfer across the blade insert in order to compare them with the equation of reference 5 which neglects heat transfer across the insert. These heat-transfer equations result in four temperature distribution equations which form the basis for the calculation of the coolant flow requirements for the return-flow blade. The four equations describe the blade shell temperature T_B, blade insert temperature T_S, coolant temperature between the blade shell and insert $T_{c,I}$, and the coolant temperature in the hollow insert $T_{c,II}$ as a function of the blade spanwise distance measured from the blade root x. (All symbols are defined in appendix A.) In the present analysis, these four equations were obtained in both the form of a difference solution and a closed-form solution. (The derivations of the temperature equations are given in appendix B.) In the derivation of the closed-form-solution equations, the coolant heat-transfer coefficient h_c is assumed constant along the blade span. In reality, however, h_c varies along the blade span because of changes in the coolant fluid properties with coolant temperature. Consequently, the required coolant flows for the return-flow blade were obtained using the difference-solution equations which permit a variation of h_c along the blade span. In this way, for those cases wherein it was determined that there was a large variation in h_c along the blade span, results from the closed-form-solution equations may be compared with results obtained using the difference-solution equations. Also, the closed-form-solution equation of reference 5 which neglects heat transfer across the blade insert may be compared with the closed-form-solution equations of the present analysis to determine the effects of neglecting heat transfer across the blade insert. (Eq. (20) of ref. 5 is given in appendix B as eq. (B21).) As in the present analysis, the temperature equation of reference 5 was derived by assuming that h_c is constant along the blade span. Further discussion of the results obtained using the difference-solution equations and closed-form-solution equations will be presented in the section RESULTS AND DISCUSSION.

In order to obtain convenient relations for the temperature distribution equations derived herein, the following simplifying assumptions were introduced:

(1) The gas-to-blade heat-transfer coefficient and effective gas temperature are constant chordwise and spanwise on the blade and are equal to the midspan value.

(2) The radial heat conduction in the blade shell and insert metal is negligible compared with the total heat transfer.

(3) Radiation is negligible.

(4) The temperature drop through the blade metal is negligible.

(5) The coolant passage flow area is constant in a spanwise direction.

In addition, one other assumption was made to simplify the coolant pressure requirement calculations for the blade considered herein. It was assumed that the 180° bend in the coolant passage at the blade tip produced no pressure loss due to the bend or the mixing of the coolant. A check was made for a few severe cases, and it is believed that the additional pressure drop due to the bend would be negligible for most of the conditions investigated, especially those having coolant inlet static pressure of 6000 and 9000 pounds per square foot.

From the difference solution, the blade shell and insert temperatures at a given spanwise position are (see appendix B)

$$T_B = \frac{T_{c,I} + \lambda_B T_{g,e}}{1 + \lambda_B} \tag{1}$$

$$T_S = \frac{T_{c,II} + \lambda_S T_{c,I}}{1 + \lambda_S} \tag{2}$$

where

$$T_{c,I} = \frac{T_{c,I,x} + T_{c,I,x+\Delta x}}{2} \tag{3}$$

and

$$T_{c,II} = \frac{T_{c,II,x} + T_{c,II,x+\Delta x}}{2} \tag{4}$$

The coolant temperatures at a given spanwise position are

$$T_{c,I,x+\Delta x} = T_{c,I,x} + \frac{h_g l_o \Delta x (T_{g,e} - T_B) + h_{c,II,S} l_S \Delta x (T_{c,II} - T_S)}{w_c c_p} +$$

$$\frac{\omega^2 \Delta x [2r_h + \Delta x (2n - 1)]}{2gJc_p} \tag{5}$$

and

$$T_{c,II,x} = T_{c,II,x+\Delta x} - \frac{h_{c,II,S} l_S \Delta x (T_{c,II} - T_S)}{w_c c_p} - \frac{\omega^2 \Delta x [2r_h + \Delta x (2n - 1)]}{2gJc_p} \tag{6}$$

The procedure for calculating the temperature distributions described by equations (1), (2), (5), and (6) is an iterative process which requires an additional equation relating the total heat input to the blade and the temperature change of the coolant:

$$Q_t = w_c c_p (T_{c,out} - T_{c,in}) = h_g l_o b (T_{g,e} - T_{B,av}) \qquad (7)$$

The calculation procedure used herein will be discussed fully in the following section, ANALYTICAL PROCEDURES. The equations derived from the closed-form solution which describe T_B, T_S, $T_{c,I}$, and $T_{c,II}$ are given by equations (B15) to (B18) in appendix B; the equation which describes T_B when the heat transfer to the insert is neglected is given as equation (B21).

The basic equations used to evaluate the pressure requirements of the recirculating-type hollow blade with insert are presented in detail in reference 10. In order to utilize the pressure equations of reference 10 in the present analysis, which employs an IBM 650 computer, the equations were solved analytically.

ANALYTICAL PROCEDURES

The cooling effectiveness of the recirculating-type hollow turbine blade with insert is indicated by the amount of coolant flow and blade-inlet coolant temperature required to maintain a given blade shell temperature and be within pressure drop limitations. The determination of the required coolant flow necessitates the evaluation of the radial blade shell temperature distribution from equation (1) or equation (B15) if the closed-form solution is used. This blade temperature distribution is obtained for an assumed coolant flow and then matched to the allowable spanwise blade temperature distribution until a point of tangency exists between the two temperature distributions at a spanwise position (see fig. 1, ref. 9). The span position where this point of tangency occurs is called the critical blade section. The allowable turbine blade temperature is determined by the design value of the turbine blade stresses and the blade material stress-rupture properties. A detailed description of this general procedure for determining the blade critical section is given in reference 9.

When using the difference-solution equations, an examination of equation (1) shows that an involved iterative solution is required in order to obtain the required coolant flow for a given set of conditions. Before equation (1) can be solved, equations (2) to (6) must be evaluated for a given spanwise increment. Before equation (1) can be applied to determine the blade shell temperature distribution, the turbine operating conditions and geometry must be specified. In addition, the

blade-inlet coolant temperature and the coolant heat-transfer coefficient in coolant legs I and II must be known. Then, an allowable spanwise blade temperature distribution must be specified in order to determine the required coolant flow. The following sections discuss the methods used to obtain the variables necessary to solve equation (1) and are followed by a discussion of the calculation procedure used in the analysis. When either of the closed-form-solution equations is used, it is necessary to solve for only T_B (eqs. (B15) or (B18)). Iteration is again required to obtain the required coolant flow for a given set of conditions.

Turbine Geometry and Operating Conditions

In order to impose severe cooling conditions on the return-flow turbine rotor blade design chosen for this analysis, an engine design representative of that required for supersonic flight at high altitudes was selected. The turbine, which is properly matched with the other engine components, operates at an inlet temperature of 3000° R and a centrifugal stress at the blade root of the single-stage rotor of 50,000 psi. It is assumed that the total metal area of the blade shell and insert supports the load. The turbine has a tip speed of about 1400 feet per second, a hub-tip radius ratio of 0.63, and a rotor blade length of 6.0 inches. Two values of average gas-to-blade heat-transfer coefficient corresponding to operation at a flight Mach number of 2.5 and altitudes of 50,000 and 80,000 feet were investigated.

Allowable Blade Temperature

For this analysis the spanwise allowable blade temperature distribution was evaluated for the 100-hour-life stress-rupture properties of the high-strength alloy, A-286, and the design stress distributions. The design spanwise stress distribution was based on a turbine blade root centrifugal stress of 50,000 psi, an area taper ratio of 0.5, and a stress-ratio factor. Stress-ratio factors of 1.5 and 2.0 were assumed for this analysis. The stress-ratio factor is a constant of proportionality meant to include the effects of other stresses, such as bending, vibration, and thermal stresses, as well as to provide some margin of safety for the effects that fabrication may have on the blade material strength (ref. 9).

Gas-to-Blade Heat-Transfer Coefficient

The average gas-to-blade heat-transfer coefficient was determined from the correlation equation and method given in reference 11. The

following equation, in the notation of the present report, results in

$$h_g = \frac{\overline{F}k_{g,B}}{(l_o/\pi)^{1-z}}\left[\left(\frac{w\sqrt{T''}}{AP''}\right)_g\left(\frac{P''}{\sqrt{T''}}\right)_g\left(\frac{T_g}{T_B}\right)\left(\frac{1}{\mu_{g,B}^g}\right)\right]^z (Pr_{g,B})^{1/3} \qquad (8)$$

In accordance with reference 11, an average Mach number of 0.70 through the blade channel was assumed, and $(w\sqrt{T''}/AP'')_g$ was evaluated from reference 10. For the turbine rotor blade considered herein, values of $\overline{F} = 0.092$ and $z = 0.70$ were assumed (see ref. 11). The values of P'' and T'' used in equation (8) were calculated at the turbine stage exit, and the Prandtl number and other gas physical properties were based on the blade temperature. For a flight Mach number of 2.5 and altitudes of 50,000 and 80,000 feet, the average gas-to-blade heat-transfer coefficients obtained from equation (7) are 0.058 and 0.0213 Btu/(sec)(sq ft) (°F), respectively. Of course, it is possible to choose other flight conditions and obtain the same values of gas-to-blade heat-transfer coefficient. For example, a heat-transfer coefficient of 0.058 Btu/(sec) (sq ft)(°F) also occurs at approximately static sea-level conditions.

Coolant Heat-Transfer Coefficient

For the type of cooled blade considered herein, the coolant flow regime may be either laminar, transitional, or turbulent. In addition, the flow is probably a mixture of forced flow and free-convection flow where the direction of the free-convection flow is opposite to that of the forced-convection flow. This type of flow is referred to as counterflow (see ref. 12). Since there is only a limited amount of heat-transfer information available for mixed-flow regimes in counterflow (ref. 12) and it appears that for conditions considered in this analysis the flow is very nearly in the forced-flow regime, forced-flow – heat-transfer correlations were used.

Heat-transfer correlations in the laminar and transitional flow regimes are affected by the geometry of the coolant passage in which the geometry is usually defined by an aspect ratio α (ratio of the longer to the shorter side of a rectangular tube). For the return-flow blade considered herein, the coolant flow between the blade shell and insert and the flow in the insert can be represented as the flow between parallel planes (i.e., a large aspect ratio). In figure 2 of reference 6, it is seen that the heat-transfer correlations for laminar, transitional, and turbulent flow can be plotted conveniently as the ratio of Nusselt number to Reynolds number Nu/Re against Reynolds number. For this analysis, it is assumed that $\alpha = 16$ (this value corresponds to flow between parallel planes). It is noted in figure 2 of reference 6

that, for $\alpha = 16$, the ratio Nu/Re varies only slightly in the transitional regime. Therefore, it was assumed that Nu/Re is constant in the transitional regime and at the same value that results from turbulent flow for a Reynolds number of 7000. This transitional regime was then assumed to extend down to a Reynolds number that would also result in the same Nu/Re ratio in laminar flow.

It must also be pointed out that the flow between the blade shell and insert is essentially the same as the flow between parallel planes heated only on one side, whereas the flow in the blade insert approaches flow between parallel planes heated equally on both sides. It is assumed that for turbulent and transitional flow regimes, the same heat-transfer correlations apply for flow between the blade shell and insert and for flow in the insert because of the flat temperature profiles observed for these two regimes. For laminar flow, however, the temperature profile requires that different heat-transfer correlations be used. Thus, for flow in the insert (equal heat input on both sides), the laminar flow heat-transfer correlation is $Nu = 7.6$ (see fig. 2, ref. 6). It can be shown from the laws of conduction that for the case of heat input on one side only, the Nusselt number for laminar flow is about one-half the value obtained for equal heat input on both sides of the parallel planes. Thus, for laminar flow between the blade shell and insert, $Nu = 3.8$. For the case when the laminar flow $Nu = 7.6$, the transitional regime exists between Reynolds numbers of 2400 and 7000, and for $Nu = 3.8$ it was assumed that the transitional regime exists between Reynolds numbers of 1200 and 7000. Thus, the heat-transfer correlations used in the present analysis are (ref. 6):

Flow regime	Reynolds number, Re	Flow between blade shell and insert, Nu	Flow in blade insert, Nu
Laminar	≤ 1200 ≤ 2400	3.8 ---	--- 7.6
Transitional	$1200 < Re < 7000$ $2400 < Re < 7000$	0.00317 Re	
Turbulent	≥ 7000	$0.023(Re)^{0.8}(Pr)^{0.4}$	

The fluid properties in the preceding heat-transfer correlations are based on a film temperature. The film temperature for any surface coefficient is defined as the average of the blade metal temperature and the bulk coolant temperature at the span position being considered. The fluid properties of hydrogen, helium, and air were obtained from references 13, 14, and 15, respectively.

Coolant Friction Coefficients

In order to determine the pressure requirements for the blade being analyzed, it is necessary that coolant friction coefficients be known. For laminar flow (Re < 2400) and turbulent flow (Re > 7000) the friction coefficients were obtained directly from figure 3 of reference 6 where for the laminar case the aspect ratio α is 16. In addition, it was assumed that the equation representing turbulent friction coefficients would also represent those occurring in the transitional regime (7000 > Re > 2400) in order to facilitate the pressure calculations. Since this will result in slightly higher friction coefficients in the transitional regime, the pressure will probably be higher than that actually required. The two equations used are

$$f = \frac{22}{Re}$$

for laminar flow, and

$$\frac{1}{\sqrt{4f}} = 2 \log \left(\sqrt{4f} \ Re \right) - 0.8$$

for turbulent flow. The turbulent-flow equation is the Kármán-Nikuradse equation (ref. 16).

Calculation Procedure

In addition to the variables specified by the turbine geometry and operating conditions, a range of blade-inlet coolant temperatures (250°, 500°, 750°, and 1000° R) and blade-inlet coolant static pressures (3000, 6000, and 9000 lb/sq ft) were specified.

In order to determine the required coolant flow within pressure drop limitations for a given coolant, blade operating conditions, and a blade coolant geometry, the coolant flow is calculated first from a heat-transfer standpoint, and then this coolant flow is used to determine the coolant pressure distribution. The calculation procedure for the heat-transfer calculations was first to divide the blade into 10 equal spanwise increments and assume a coolant flow. (For a few cases, 20 equal increments were assumed. This resulted in essentially the same temperature distributions and a required coolant flow about 2 percent lower than the one obtained with 10 increments.) Then, equation (7) was used to obtain an approximate value of the total heat input to the blade where the average value of the allowable blade temperature distribution was used as the average blade temperature.

Using the assumed coolant flow and the approximate total heat input, the blade-outlet coolant temperature was determined from equation (7). Then, equations (2), (5), (4), (1), (6), (3), and (1) were applied to the first increment (at the blade root) in the order given. In this case, the blade-inlet and -outlet coolant temperatures were used in equation (2) for the average values, and the blade-inlet coolant temperature was used the first time equation (1) is referred to so that an approximate value of blade temperature was determined. The final use of equation (1) gives the average blade shell temperature at the midpoint of the first increment. At this point, the average coolant temperature as obtained from equations (5) and (6) was used to recalculate the temperature values for the first increment. The calculation proceeds to the second increment by utilizing the temperature values obtained for the first increment.

This procedure continues until a spanwise temperature distribution has been obtained for the assumed coolant flow. The calculated blade temperature distribution is compared to the allowable temperature distribution. If no point of tangency exists between the two curves, another coolant flow is assumed until a point of tangency does occur. Then the average blade temperature for the coolant flow which gives a point of tangency is used in equation (7) to determine the total heat input to the blade. The entire procedure is repeated until a heat balance is obtained for the blade. That is, the heat flow into the blade is equal to the product of the flow rate and enthalpy rise of the coolant. This final value of coolant flow is then used in the pressure distribution equations described in reference 10, and the pressure change through the blade is determined.

The calculation procedure previously described for the difference-solution equations would apply if the closed-form-solution equations were used. The advantage of using the closed-form-solution equations is that an individual temperature distribution such as T_B (eq. (B15) or (B19)) may be obtained without regard to the other temperature distributions. In addition, the closed-form-solution equations offer the possibility of calculating a temperature at a given spanwise position without calculations at the preceding spanwise positions.

RESULTS AND DISCUSSION

Comparison of Temperature Distributions Obtained from Difference-Solution and Closed-Form-Solution Equations

The spanwise variations in blade shell, coolant, and blade insert temperatures obtained using the difference-solution equations and the

two closed-form-solution equations are shown in figure 3. Equations (1) to (6) were used for the difference solution, equations (B15) to (B18) were used for the closed-form solution considering heat transfer across the blade insert, and equation (B21) was used for the closed-form solution neglecting heat transfer across the blade insert. The results shown in figure 3 are for a spacing of 0.020 inch between the blade shell and insert, a gas-to-blade heat-transfer coefficient of 0.058 Btu/(sec)(sq ft)($^\circ$F), and a stress-ratio factor of 1.5. Figure 3(a) presents the results obtained for air and a coolant inlet temperature of 1000° R having a required coolant flow of 0.1235 pound per second. The results shown on figure 3(b) are for hydrogen and a coolant inlet temperature of 250° R having a required coolant flow of 0.0099 pound per second. The temperature distributions illustrated on figure 3 are typical of those obtained for the other conditions investigated, the only difference being that the temperature gradients may be more or less than those indicated. A more detailed discussion of the trends observed on figure 3 will be given in the subsequent section Illustrative Spanwise Temperature Distributions.

As pointed out previously, in the solution of the closed-form-solution equations it is assumed that the inside heat-transfer coefficient h_c (and thus, λ_B) is constant along the blade span. Consequently, the two cases shown in figure 3 were chosen because they have widely different variations in h_c. For the conditions of figure 3(a), the variation in h_c is approximately 200 percent. In addition to the results shown in figure 3, comparisons between the difference-solution equations and the closed-form-solution equations were also made for conditions having variations in h_c less than 25 percent and between 25 and 200 percent. The results of these comparisons are essentially the same as those indicated in figure 3. For the temperature distributions shown, a value of λ_B at a spanwise position x/b equal to 0.45 was chosen arbitrarily for the closed-form-solution equations.

In referring to figure 3(a) wherein the blade shell temperature obtained from the difference solution (eq. (1)) rises from the blade root to the tip, the shell temperature obtained from equation (B15), which considers heat transfer across the blade insert, is in fairly good agreement with the shell temperature from equation (1) except near the blade root where it is about 30° lower. At an x/b of 0.45 there is exact agreement as expected. Thus, the difference between the solid line (eq. (1)) and the dashed line (eq. (B15)) for blade shell temperatures is due to the spanwise variation in h_c. Similar comparisons between the difference solution and the closed-form solution considering heat transfer across the blade insert are indicated by the coolant and insert temperature distributions shown in figure 3(a). For the coolant temperature distribution in leg I, there is almost exact agreement between the difference and closed-form solutions.

Now, considering the blade shell temperature distribution for the closed-form solution neglecting heat transfer across the insert (eq. (B21)), it is noted that both closed-form solutions agree at the blade root and then diverge along the blade span. They agree at the blade root because the same boundary condition was applied at the blade root in the solution of the differential equations. Thus, it appears that neglecting the heat transfer across the blade insert will result in blade shell temperatures lower than when the heat transfer is considered. Also, the lower shell temperatures result in lower coolant temperatures in leg I. (The coolant temperature distribution in leg I when heat transfer was neglected across the insert was obtained using the shell temperatures from equations (B21) and (1)).

When blade shell temperature obtained from equation (1) decreases from the blade root until about an x/b of 0.6 and then increases slightly to the tip (fig. 3(b)), agreement between the difference and closed-form solutions is poor. It must be pointed out that the use of a constant value of λ_B in the closed-form-solution equations can only result in a shell temperature distribution that increases from root to tip. Regardless of the poor agreement between the blade shell temperature distributions, there is excellent agreement between the coolant and insert temperature distributions obtained with the difference- and closed-form-solution equations. The coolant temperature distribution obtained by using equations (B21) and (1) coincides with that of the closed-form solution considering heat transfer across the insert. It should also be pointed out that blade shell temperature distributions such as shown in figure 3(b) which do not increase from root to tip were obtained only for coolant inlet temperatures of 500° R or less and for turbulent or transitional flow regimes.

For a given blade design and operating conditions, turbine-cooling analyses usually involve the calculation of the required coolant flow, as was done in the present analysis, or the calculation of blade shell and coolant temperature distributions for a given coolant flow. The preceding discussion indicates that the closed-form-solution equations considering heat transfer across the blade insert may be used for the blade design considered having coolant inlet temperatures above 500° R. The choice of the spanwise position where a constant value of h_c is chosen is arbitrary. However, it is suggested that the spanwise position chosen would be the one where the blade critical section is expected to be. Depending on the degree of accuracy desired, the closed-form-solution equations neglecting heat transfer across the blade insert may be used for coolant inlet temperatures above 500° R. The use of this closed-form solution may result in blade shell temperatures about 1 to 5 percent lower than those obtained from the difference-solution equations or the closed-form-solution equations which consider heat transfer across the blade insert.

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    When the coolant inlet temperature is less than 500° R for the blade
of the present analysis, the difference-solution equations should always
be used when calculating a required coolant flow. If temperature dis-
tributions are desired for a given coolant flow wherein the coolant is
in the laminar regime, the closed-form-solution equations may be used.
For turbulent or transitional flow regimes, the difference-solution
equations should be used.

                            Tabulated Results

    Table I summarizes the computed solutions of required coolant flow
per blade (obtained using the difference-solution equations) and the re-
sulting static-pressure change through the blade for the return-flow
turbine rotor blade with a spacing between the blade shell and insert
of 0.050 inch. All results shown in table I are for a turbine-inlet
temperature of 3000° R, a turbine rotor blade root stress of 50,000 psi,
a stress-ratio factor of 1.5, and a flight Mach number of 2.5. In ad-
dition, two gas-to-blade heat-transfer coefficients are indicated. At
a flight Mach number of 2.5, the lower value of coefficient represents
an altitude of 80,000 feet, and the higher value represents an altitude
of 50,000 feet. Because a large number of calculations are involved in
obtaining each coolant flow and pressure change, only those calculations
involving a blade coolant geometry having a spacing of 0.050 inch be-
tween the blade shell and insert were completed.

    The present analysis was intended to show methods of calculation
and some of the related effects of coolant inlet temperature on blade
shell temperature distributions and coolant flow requirements for a
return-flow blade using hydrogen, helium, or air as the coolant. For a
return-flow blade in which the coolant circulates, the coolant flow re-
quirements are not as important with regard to engine performance as
those of an air-cooled blade discharging the air at the blade tip. The
most important point is to determine whether or not the coolant will cir-
culate. This requires a coolant Mach number less than 1.0 and a coolant
pressure rise in the blade (if there is no external pump). Then, the
required coolant flow must be used to determine the design and size of
the ducting necessary to transport the coolant to and from the turbine.
If a heat exchanger is used in the system, then the required coolant
flow will also affect the design of the heat exchanger.

    The required coolant flows indicated in table I are the coolant
flows per blade. As a matter of interest, the required coolant flow
per blade for an air-cooled corrugated-insert turbine rotor blade dis-
charging the air at the blade tip is about 0.05 pound per second for a
coolant inlet temperature of 1000° R and a gas-to-blade heat-transfer
coefficient of approximately 0.058 Btu/(sec)(sq ft)(°F). This value

of coolant flow is about 1/6 of that required for the air-cooled return-flow blade (table I(c)). It must be remembered, however, that the amount of coolant flow required for the return-flow blade is not as important as for the air-cooled corrugated-insert blade, because the air or other coolant does not have to be bled from the compressor flow and is not dumped into the tailpipe to dilute the exhaust gas.

An examination of table I shows that, in general, the required coolant flow decreases and then increases as the coolant inlet temperature is reduced from 1000° to 250° R. Also noted in table I are the coolant flow regimes obtained for the coolant flowing between the blade shell and insert. It is of interest to observe that laminar flow existed only for a coolant inlet temperature of 250° R, a gas-to-blade heat-transfer coefficient of 0.0213 Btu/(sec)(sq ft)($^\circ$F), and hydrogen. For all other conditions the flow was either transitional or turbulent with the majority of the results being in the transitional regime. The reasons for the trends observed in table I will be discussed in conjunction with the discussion of the figures which follow subsequently.

Pressure requirements. - The coolant static pressures at the blade inlet, tip, and outlet are listed in table I. The cases for which a coolant Mach number of 1.0 was obtained in the coolant passage are indicated as having no solution. As was pointed out previously, no correction was made of the coolant static pressure at the blade tip to account for the fact that the coolant is turned 180° at the tip. A requirement for the coolant to pass through the blade without an external pump is that there is a pressure rise (due to rotation) through the system.

When using hydrogen as the coolant (table I(a)), it is noted that there is sufficient static pressure available to pass the coolant through the blade except for a gas-to-blade heat-transfer coefficient of 0.0580 Btu/(sec)(sq ft)($^\circ$F) and a coolant inlet static pressure of 3000 pounds per square foot. An inlet pressure of 9000 pounds per square foot corresponds approximately to the compressor-exit pressure for the turbojet engine under investigation. For inlet pressures of 6000 and 9000 pounds per square foot, static-pressure rises occur through the blade for all conditions and for some of the conditions at an inlet pressure of 3000 pounds per square foot. For those cases where a pressure drop occurs through the blade for an inlet pressure of 3000 pounds per square foot, the pressure drop is small. Thus, there appears to be no major problem in having sufficient pressure aviable for passing hydrogen through the return-flow blade of this analysis.

It is noted in table I(b) (helium) that at a gas-to-blade heat-transfer coefficient of 0.0580 Btu/(sec)(sq ft)($^\circ$F) an inlet coolant pressure of 3000 pounds per square foot is insufficient to pass the required coolant flow for all coolant inlet temperatures. For coolant inlet temperatures of 750° and 1000° R, an inlet pressure of 6000 pounds

per square foot is insufficient. At the lower gas-to-blade heat-transfer coefficient all values of inlet pressure are adequate except for coolant inlet temperatures of 750° and 1000° R. Thus, the problem of passing the required amount of coolant using helium is more difficult than that using hydrogen for all the conditions investigated.

When air is used as the coolant (table I(c)), the problem of passing the required coolant flow at the high value of the gas-to-blade heat-transfer coefficient becomes much more critical than for either hydrogen or helium. A coolant inlet static pressure of at least about 9000 pounds per square foot is required for coolant inlet temperatures of 250°, 500°, and 750° R, and a pressure somewhat greater than 9000 pounds per square foot is required for an inlet temperature of 1000° R. At the lower value of the gas-to-blade heat-transfer coefficient, all values of inlet pressure are sufficient for passing the flow except for an inlet temperature of 1000° R and an inlet pressure of 3000 pounds per square foot.

From the preceding discussion it is evident that once the required coolant flow is determined for a return-flow turbine blade using a given gaseous coolant, it is extremely important that the pressure change through the blade be determined to see that the coolant can be passed.

Illustrative spanwise temperature distributions. - Two different types of blade shell temperature distribution were observed when figure 3 was referred to previously. The blade shell temperature increased from the blade root to the tip (fig. 3(a)), decreased from the blade root to about an x/b of 0.6, and then increased slightly to the blade tip (fig. 3(b)). The temperature distributions illustrated in figure 3 are fairly typical of those obtained for the other conditions investigated, with slight variations. In a few cases, the blade shell temperature distribution was practically constant from blade root to tip. For all the conditions investigated wherein the blade shell temperature decreased from blade root to tip or was practically constant, the coolant flow regime was either transitional or turbulent, and the coolant inlet temperature was 250° or 500° R. For those cases where the blade shell temperature rose from blade root to tip for coolant inlet temperatures of 250° and 500° R the coolant flow was laminar. The blade shell temperature always increased from blade root to tip regardless of the coolant flow regime for coolant inlet temperatures of 750° and 1000° R.

The decreasing blade shell temperatures from blade root to tip for coolant inlet temperatures of 250° and 500° R and the transitional or turbulent flow can be attributed to the fact that the coolant heat-transfer coefficient is a function of the bulk coolant temperature and film temperature at a given span position. The bulk coolant temperature is defined as the average of the coolant temperatures at the inlet and outlet of a blade spanwise increment, and the film temperature is the average of the blade shell and bulk temperatures at a given spanwise increment. The preceding statements will be explained somewhat by the following equations.

For turbulent flow,

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.4} \tag{9}$$

If Prandtl number is assumed constant, equation (9) can be rewritten as

$$\frac{h_c D_h}{k} = C\left(\frac{w_c}{A} \frac{D_h}{\mu g} \frac{T_b}{T_f}\right)^{0.8} \tag{10}$$

From references 13 to 15 the fluid properties can be evaluated as a function of the film temperature so that equation (10) becomes

$$h_c = \frac{C_3 w_c^{0.8} T_b^{0.8}}{T_f^s} \tag{11}$$

where s is an exponent which, depending on the coolant physical proper-ties, will vary between about 0.50 and 0.75. An equation similar to equation (11) can be obtained for the transition regime:

$$h_c = C_4 \frac{w_c T_b}{T_f^m} \tag{12}$$

where m is an exponent dependent on the coolant physical properties. The exponent m will not necessarily have the same value as s of equation (11) because of the different exponents on the Reynolds number for transitional and turbulent flows.

The coolant heat-transfer coefficient in equations (11) and (12) can be evaluated as a function of bulk temperature by assuming a constant blade temperature at any given spanwise increment and a constant coolant weight flow. At the lower bulk temperatures, 250° to 500° R, the coolant heat-transfer coefficient increases with an increase in bulk temperature at a faster rate than it does in the range of bulk temperatures above 500° R. Thus, because of the greater rate of increase of the coolant heat-transfer coefficient along the blade span in the range of low cool-ant bulk temperatures, the blade shell temperature decreases or remains relatively constant from blade root to tip when the flow is in either transitional or turbulent range.

Several Factors Affecting Required Coolant Flow

The changes in the required coolant flow due to variations in the blade-inlet coolant temperature are shown in figures 4 to 6. In addi-tion, the effects of the coolant used, the spacing between the blade

shell and insert, and the stress-ratio factor on the required coolant flow are shown in figures 4, 5, and 6, respectively.

Effect of blade-inlet coolant temperature. - An interesting point derived from figures 4 to 6 is the trend observed for the required coolant flow as the blade-inlet coolant temperature is changed. In all cases the required coolant flow decreases as the coolant inlet temperature is reduced until a minimum value is reached. Then, further reductions in the coolant inlet temperature cause an increase in the required coolant flow. Similar trends may be observed in table I. Thus, the results shown in figures 4 to 6 and table I indicate that there is some limiting value of blade-inlet coolant temperature where further reductions in this temperature are undesirable for the purpose of reducing the required coolant flow. For all the conditions investigated wherein the required coolant flow decreases and then increases with reductions in the coolant inlet temperature, the coolant is in the turbulent or transitional regime for each coolant inlet temperature (see table I). If laminar flow existed for each coolant inlet temperature, then reductions in coolant inlet temperature would cause reductions in the required coolant flow.

The trends shown in figures 4 to 6 and table I may be explained in two ways. In order to proceed with the explanation it must be pointed out that for some of the conditions investigated, the amount of heat removed Q was relatively constant as the coolant inlet temperature was changed from 1000° to 250° R. In other cases, however, the value of Q decreased and then increased as the coolant inlet temperature was reduced. Thus, for the latter cases, the required coolant flow may be higher at a coolant inlet temperature of 250° R than at, say, 500° R, because Q is higher at 250° R. For the cases wherein Q is relatively constant, the trends shown in figures 4 to 6 and table I may be explained by using equations (11) and (12) and the following relation:

$$Q = h_c l_i b (T_B - T_b) \tag{13}$$

Combining equations (11) and (13) and equations (12) and (13), solving for the coolant flow, and assuming that Q is constant result in

$$w_c^{0.8} = \frac{c_5 \left(\dfrac{T_B + T_b}{2} \right)^s}{T_b^{0.8}(T_B - T_b)} \tag{14}$$

for turbulent flow, and

$$w_c = \frac{c_6 \left(\dfrac{T_B + T_b}{2} \right)^m}{T_b(T_B - T_b)} \tag{15}$$

for transitional flow. Assuming a range of values of blade and bulk temperatures for coolants of hydrogen, helium, and air for equations (14) and (15) shows exactly the same trends as those in figures 4 to 6 and table I. That is, as the bulk temperature was reduced, the coolant flow decreased until a minimum was reached and then increased with further reductions in bulk temperature. This again is due to the effect of the coolant properties (expressed as a function of film temperature) on the coolant heat-transfer coefficient. The effect of variations in blade temperature shifted the minimum temperature point: the higher the blade temperature, the higher the minimum coolant temperature.

Effect of coolant. - The effect of the type of coolant used in the return-flow turbine blade on the required coolant flow is illustrated in figure 4 for a gas-to-blade heat-transfer coefficient of 0.0580 Btu/(sec)(sq ft)(^{O}F). As would be expected because of the excellent heat-transfer characteristics of helium and hydrogen (the specific heats of helium and hydrogen are 1.24 Btu/(lb)(^{O}F) and about 3.5 Btu/(lb)(^{O}F), respectively, as compared to 0.24 Btu/(lb)(^{O}F) for air), the required coolant flows have been reduced considerably when using helium or hydrogen instead of air. For example, the use of air as the coolant for a given coolant inlet temperature requires an increase in the required coolant flow of about 15 times over that required when using hydrogen and about 6 times as much as that required when using helium. It is noted that the ratio of the coolant flow requirements for any two of the coolants is approximately the same as the ratio of their specific heats.

Effect of spacing between blade shell and insert. - The reductions obtained in the required coolant flow as the spacing between the blade shell and insert is decreased from 0.070 to 0.020 inch are presented for a gas-to-blade heat-transfer coefficient of 0.0580 Btu/(sec)(sq ft)(^{O}F) using coolants of hydrogen (fig. 5(a)) and air (fig. 5(b)). For both hydrogen and air the required coolant flow is reduced by a factor of approximately $3\frac{1}{2}$ as the spacing is changed from 0.070 to 0.020 inch. The coolant is in the turbulent or transitional regime for all of the conditions shown in figure 5.

From a heat-transfer standpoint, the advantage of decreasing the spacing between the blade shell and insert from 0.070 to 0.020 inch is apparent in figure 5. For both hydrogen and air the required coolant flow has been reduced considerably. For example, the required coolant flow for a spacing of 0.070 inch, hydrogen, and a coolant inlet temperature of about 600° R is approximately 4 times the amount required for a spacing of 0.020 inch with other conditions being the same. It was expected that the coolant flow requirements would be reduced as the spacing between the blade shell and insert was changed from 0.070 to 0.020 inch because the coolant velocities are increased as the flow passage becomes smaller and, thus, the coolant heat-transfer coefficient increases.

The preceding discussion indicates that, from a fabrication standpoint, it is desirable to have as small a spacing between the blade shell and insert as practical in order to reduce the coolant flow requirements. However, the results shown in figure 5 do not consider the pressure limitations that may be encountered for the various spacings and coolants shown. It is noted in table I(a) for a gas-to-blade heat-transfer coefficient of 0.0580 Btu/(sec)(sq ft)($^\circ$F) that sufficient pressure is available for passing the coolant flow for all coolant inlet temperatures and pressures except for a coolant inlet temperature of 1000° R and a pressure of 3000 pounds per square foot. Although the results shown in table I(a) are for hydrogen and a spacing of 0.050 inch, it may be assumed that there will be sufficient pressure available to pass the required coolant flows obtained for hydrogen, a spacing of 0.070 inch, and the conditions indicated in table I(a). This assumption cannot be made when the spacing is reduced to 0.020 inch. For the conditions indicated in figure 5(a) and a spacing of 0.020 inch, calculations show that a coolant inlet pressure of 3000 pounds per square foot was not sufficient to pass the coolant flow at any coolant inlet temperature, and a coolant inlet pressure of 6000 pounds per square foot was not sufficient to pass the coolant flow at coolant inlet temperatures of 750° and 1000° R. It is noted in table I(c) and figure 5(b) for a spacing of 0.050 inch that with air as the coolant a coolant inlet pressure of 9000 pounds per square foot is required to pass the flow for inlet temperatures of 250°, 500°, and 750° R, and a value greater than 9000 pounds per square foot would probably be required to pass the flow at a coolant inlet temperature of 1000° R. For a spacing of 0.070 inch with air as the coolant, the pressure requirements were only slightly better than those indicated in table I(c), and for a spacing of 0.020 inch the pressure requirements were worse. Thus, in order to design successfully a recirculating-type hollow turbine blade with insert, both heat-transfer and pressure calculations must be made for the given coolant geometry, coolant, and conditions.

Effect of stress-ratio factor. - The design value of the turbine rotor blade stress is usually larger than the actual centrifugal stress for cooled turbine blades. This difference may be accounted for by introducing a constant of proportionality called the stress-ratio factor. Limited unpublished experimental data obtained on air-cooled turbine rotor blades fabricated from high-temperature alloys indicate that stress-ratio factors from about 1.5 to 2.0 were obtained. The results that have been presented in this report have been for a stress-ratio factor of 1.5. In order to determine the effect of a stress-ratio factor of 2.0 on the required coolant flow, calculations were made for hydrogen, a spacing between the blade shell and insert of 0.050 inch, and a gas-to-blade heat-transfer coefficient of 0.0580 Btu/(sec)(sq ft) ($^\circ$F). The results of this calculation are shown in figure 6. With a large stress-ratio factor the allowable blade temperature for a given

stress value will be decreased and, thus, the coolant flow requirements
will be higher. The results shown in figure 6 indicate that the required
coolant flow for a stress-ratio factor of 2.0 is about 12 to 20 percent
higher than that obtained for a stress-ratio factor of 1.5.

Effect of coolant flow rate on size of coolant ducting. - Once it
has been established that for a given set of conditions and return-flow
blade geometry the required coolant flow can be circulated, the size of
ducting necessary to transport the coolant to and from the turbine must be
determined. If a heat exchanger is used in the coolant system, then the
required coolant flow will also affect the design of the heat exchanger.
The calculations necessary to determine the ducting size and possible
heat-exchanger design for the return-flow blade cooling system are be-
yond the scope of this report. However, it is desirable to have some
indication of the relative size or weight of the ducting required to
transport hydrogen, helium, or air with all other operating conditions
remaining the same.

A comparison of the size of the ducting required to transport the
coolant to the blade was made on the basis of equal pressure losses for
a condition of turbulent flow in the ducts. This comparison was made in
the following manner:

The pressure loss in the pipe due to friction alone can be expressed
as

$$\Delta P = \frac{2\rho v^2 fL}{gD} \tag{16}$$

for the case of constant values of pressure loss $\Delta P/P$, length L, and
coolant Mach number M. Equation (16) can be rewritten as

$$D = K_1 f \, \Upsilon \tag{17}$$

For turbulent flow the friction coefficient can be given by (ref. 17,
p. 119)

$$f = \frac{0.046}{Re^{0.2}} \tag{18}$$

Combining equations (17) and (18) gives

$$D = K \frac{\Upsilon}{Re^{0.2}} \tag{19}$$

If the Reynolds number of the coolant is defined as

$$Re = \frac{w_c D}{A \mu g}$$

then equation (19) becomes

$$D = K_3(\gamma)^{1.25}\left(\frac{\mu}{w_c}\right)^{0.25} \tag{20}$$

Using equation (20) and the properties of the coolant at the blade inlet, the required coolant ducting diameters (pipe diameters) are presented in figure 7 on a relative bases where the required pipe diameter for air at an inlet temperature of 250° R was used as the base. The conditions investigated are for a gas-to-blade heat-transfer coefficient of 0.058 Btu/(sec)(sq ft)($^\circ$F).

Figure 7 shows that the use of air as the coolant would require the smallest size ducting. Hydrogen and helium require pipes about 1.6 and 1.9 times the diameter of the air ducting. Helium requires a larger size pipe than hydrogen because of the effect of a larger value of γ in equation (20). The values of γ for helium, hydrogen, and air are approximately 1.66, 1.39, and 1.40, respectively. The air curve stops at an inlet temperature of 750° R because choking occurred in the coolant passage for an inlet temperature of 1000° R (see table I(c)).

Without a complete analysis not too much can be said about the weight of the required ducting except that the pipe weight will increase with the required diameter and wall thickness, which will be a function of the required pressure for constant $\Delta P/P$.

The preceding discussion points out the necessity of taking extreme care in evaluating a return-flow blade coolant system using hydrogen, helium, or air. Although the required coolant flow rates for hydrogen and helium are considerably smaller than those obtained for air, the coolant ducting requirements based on equal pressure losses, equal coolant Mach number, and equal duct strengths are more severe than those for air with respect to size and weight.

CONCLUDING REMARKS

The purpose of this investigation was to obtain the coolant flow and pressure requirements for a return-flow turbine rotor blade design which utilizes hydrogen, helium, or air as the coolant. The results presented herein were obtained for only one turbine geometry and two

operating conditions which are representative of a turbojet engine capable of supersonic flight at high altitudes. In addition, a range of coolant inlet temperatures and pressures was assumed. It is believed that the trends observed in this analysis are representative of those which would be obtained at other operating conditions.

Without further study it is difficult to reach definite conclusions as to the relative advantages or disadvantages of air-cooling systems wherein the air is discharged at the blade tip, liquid-cooling systems, and the recirculating-gas cooling system considered in the present report. For a given engine design and operating conditions each cooling system must be evaluated on the basis of (1) ease of fabrication of the particular cooled turbine blade design, (2) the required size and weight of the coolant ducting, and (3) the effect of the given cooling system on engine performance. With regard to engine performance, the recirculating-gas cooling system would have the same effect on performance as the liquid-cooling system.

On the basis of the results of this analysis it appears that the return-flow blade design considered herein would cool turbine blades adequately over a wide range of operating conditions using either hydrogen, helium, or air as the coolant. It is well to point out that cooling systems using hydrogen or helium may present additional complications because of the difficulty in sealing these two coolants in a closed system. Also, on the basis of the preliminary calculations made on coolant duct size, both hydrogen and helium will require larger and possibly heavier coolant ducts than air, even though their respective coolant flows are considerably less than air.

SUMMARY OF RESULTS

The results of this analytical investigation to determine the coolant flow and pressure requirements of a return-flow turbine blade design which utilized hydrogen, helium, or air are summarized as follows:

1. The use of air as the coolant for the return-flow blade requires an increase in the required coolant flow of about 15 times that required when using hydrogen and about 6 times as much as that required when using helium.

2. When air is used as the coolant for the return-flow blade, the problem of passing the required coolant flow becomes much more critical than for either hydrogen or helium.

3. Regardless of the coolant used, with transitional or turbulent flow a minimum value of coolant inlet temperature is reached where further reductions in this temperature will result in increases in the required coolant flow.

4. Decreasing the spacing between the blade shell and insert from 0.070 to 0.020 inch reduced the required coolant flow by a factor of approximately $3\frac{1}{2}$ for hydrogen and air used as the coolants. Care must be taken, however, that the spacing is not so small that extremely high pressures will be necessary to pass the required coolant flow.

5. For a range of coolant inlet temperatures from 250° to 1000° R, a change in stress-ratio factor from 1.5 to 2.0 results in about a 12 to 20 percent increase in the required coolant flow.

6. On the basis of preliminary calculations, the use of air as the coolant for the return-flow blade would require the smallest diameter coolant ducting to transport the coolant to and from the turbine blades. Hydrogen and helium required pipes about 1.6 to 1.9 times the diameter of the air ducting.

7. The use of equations which neglect the heat transferred across the blade insert will result in blade shell temperatures from about 1 to 5 percent lower than those obtained when the heat transfer across the insert is considered.

Lewis Flight Propulsion Laboratory
      National Advisory Committee for Aeronautics
            Cleveland, Ohio, February 21, 1957

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APPENDIX A

SYMBOLS

A   flow area, sq ft

b   turbine blade span or length, ft

C   constant in eq. (10)

$\left.\begin{array}{c} C_3, C_4 \\ C_5, C_6 \end{array}\right\}$ constants in eqs. (11), (12), (14), and (15)

c_p specific heat at constant pressure, Btu/(lb)($^\circ$F)

D   diameter, ft

D_h hydraulic diameter (for infinite slot, 2 times spacing which makes up flow passage), ft

$\overline{F}$ mean coefficient, function of blade transition ratio and Euler number

f   friction coefficient

g   standard acceleration due to gravity, 32.174 ft/sec^2

h   average heat-transfer coefficient, Btu/(sec)(sq ft)($^\circ$F)

J   mechanical equivalent of heat, 778 ft-lb/Btu

$\left.\begin{array}{c} K_1, K_2 \\ K_3 \end{array}\right\}$ constants in eqs. (17), (19), and (20)

k   thermal conductivity, Btu/(sec)(ft)($^\circ$F)

L   length, ft

l perimeter

M   coolant Mach number

m   exponent, function of coolant physical properties

n   number of blade spanwise increments

Nu   Nusselt number, $h_c D_h / k_c$

P   static pressure, lb/sq ft

P"   total pressure relative to turbine rotor blade, lb/sq ft

Pr   Prandtl number, $c_p \mu g / k$

Q   heat input, Btu/sec

Re   Reynolds number of coolant, $w_c D_h / A \mu g$

r   radius, ft

s   exponent, function of coolant physical properties

T   temperature, $^\circ R$

T"   total temperature relative to turbine rotor blade, $^\circ R$

T_B average blade shell temperature at spanwise position, $^\circ R$

T_b bulk coolant temperature, $^\circ R$

T_c coolant temperature with respect to coolant passage, $^\circ R$

T_f film coolant temperature, $^\circ R$

v   velocity, ft/sec

w   weight flow, lb/sec

x   spanwise distance measured from blade root, ft

z   exponent, function of blade transition ratio and Euler number

α aspect ratio, longer side of rectangular tube/shorter side of tube

γ ratio of specific heats

λ_B $h_g/h_{c,I,B}$

λ_S $h_{c,I,S}/h_{c,II,S}$

μ viscosity, slug/(sec)(ft)

ρ density, lb/cu ft

ω angular velocity, radian/sec

Subscripts:

av   average

B   blade shell

c   coolant

e   effective

g   gas

h   blade root

i   inside

in   blade inlet

n   number of spanwise increments

o   outside

out   blade outlet

r   radius, ft

S   blade insert

t   total

x   spanwise distance measured from blade root

I   coolant leg between blade shell and blade insert (see fig. 2)

II   coolant leg in hollow insert (see fig. 2)

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APPENDIX B

ONE-DIMENSIONAL SPANWISE TEMPERATURE DISTRIBUTION EQUATIONS FOR

RECIRCULATING-TYPE HOLLOW TURBINE BLADE WITH INSERT

Difference Solution Considering Heat Transfer Across Insert

A heat balance for a small section of blade height Δx (see fig. 2), neglecting radial conduction and radiation, results in the following two equations:

$$h_g l_o \Delta x (T_{g,e} - T_B) = h_{c,I,B} l_o \Delta x (T_B - T_{c,I}) \tag{B1}$$

and

$$h_{c,II,S} l_S \Delta x (T_{c,II} - T_S) = h_{c,I,S} l_S \Delta x (T_S - T_{c,I}) \tag{B2}$$

The temperature rise of the coolant in leg I due to heat transferred by the shell and insert is

$$w_c c_p (T_{c,I,x+\Delta x} - T_{c,I,x}) = h_g l_o \Delta x (T_{g,e} - T_B) + h_{c,II,S} l_S \Delta x (T_{c,II} - T_S) \tag{B3}$$

The heat transferred by the insert to the coolant in leg II is

$$w_c c_p (T_{c,II,x+\Delta x} - T_{c,II,x}) = h_{c,II,S} l_S \Delta x (T_{c,II} - T_S) \tag{B4}$$

The coolant temperature rise from the blade root $(r = r_h)$ to any radius r due to compression is given by

$$\frac{dT_c}{dr} = \frac{\omega^2 r}{gJc_p} \tag{B5}$$

where

$$r = r_h + x$$

Integrating equations (B5) between the limits of r_h and r gives

$$T_{c,r} - T_{c,r_h} = \frac{\omega^2 (r^2 - r_h^2)}{2gJc_p} \tag{B6}$$

By considering n number of increments Δx, equation (B6) can be written as

$$T_{c,r+n\Delta x} - T_{c,r_h+(n-1)\Delta x} = \frac{\omega^2}{2gJc_p} \left\{ (r_h + n\Delta x)^2 - [r_h + (n-1)\Delta x]^2 \right\}$$

or, by simplifying,

$$T_{c,r+n\Delta x} - T_{c,r_h+(n-1)\Delta x} = \frac{\omega^2 \Delta x}{2gJc_p} [2r_h + \Delta x(2n-1)] \qquad (B7)$$

By combining equations (B3) and (B7), the coolant temperature out in leg I for the increment Δx (fig. 2) becomes

$$T_{c,I,x+\Delta x} = T_{c,I,x} + \frac{h_g l_o \Delta x(T_{g,e} - T_B) + h_{c,II,S} l_S \Delta x(T_{c,II} - T_S)}{w_c c_p} +$$
$$\frac{\omega^2 \Delta x [2r_h + \Delta x(2n-1)]}{2gJc_p} \qquad (5)$$

Likewise, by combining equations (B4) and (B7) the coolant temperature out in leg II for the increment Δx (fig. 2) becomes

$$T_{c,II,x} = T_{c,II,x+\Delta x} - \frac{h_{c,II,S} l_S \Delta x(T_{c,II} - T_S)}{w_c c_p} - \frac{\omega^2 \Delta x [2r_h + \Delta x(2n-1)]}{2gJc_p}$$
$$(6)$$

The blade shell and insert temperatures for an increment Δx are obtained from equations (B1) and (B2):

$$T_B = \frac{T_{c,I} + \lambda_B T_{g,e}}{1 + \lambda_B} \qquad (1)$$

$$T_S = \frac{T_{c,II} + \lambda_S T_{c,I}}{1 + \lambda_S} \qquad (2)$$

where

$$\lambda_B = \frac{h_g}{h_{c,I,B}}$$

$$\lambda_S = \frac{h_{c,I,S}}{h_{c,II,S}}$$

Closed-Form Solution Considering Heat Transfer Across Blade Insert

A closed-form solution may be obtained by utilizing equations (1) and (2) and the following two expressions for the change in coolant temperature in legs I and II due to heat transfer and compression:

$$\frac{dT_{c,I}}{dx} = \frac{h_g l_o (T_{g,e} - T_B)}{c_p w_c} + \frac{h_{c,II,S} l_S (T_{c,II} - T_S)}{c_p w_c} + \frac{\omega^2 (r_h + x)}{gJc_p} \tag{B8}$$

$$\frac{dT_{c,II}}{dx} = \frac{h_{c,II,S} l_S (T_{c,II} - T_S)}{c_p w_c} + \frac{\omega^2 (r_h + x)}{gJc_p} \tag{B9}$$

The next step in the derivation of the desired blade and insert temperature distribution equations is to solve equation (1) for $T_{c,I}$, obtain $dT_{c,I}/dx$, solve equation (2) for $T_{c,II}$, and obtain $dT_{c,II}/dx$. The value of $T_{c,I}$ obtained from equation (1) is inserted in equation (2). In addition, the value of $T_{c,II}$ obtained from equation (2) is inserted in equations (B8) and (B9). Therefore, by assuming $T_{g,e}$ and λ_B are constant along the blade span, the following is obtained from equation (1):

$$\frac{dT_{c,I}}{dx} = (1 + \lambda_B) \frac{dT_B}{dx} \tag{B10}$$

From equations (1) and (2), for constant values of $T_{g,e}$, λ_B, and λ_S

$$\frac{dT_{c,II}}{dx} = (1 + \lambda_S) \frac{dT_S}{dx} - \lambda_S (1 + \lambda_B) \frac{dT_B}{dx} \tag{B11}$$

Equating equations (B8) and (B10), substituting the value of $T_{c,II}$ from equation (2) (and substituting the value of $T_{c,I}$ from eq. (1) in eq. (2)) in equation (B8), and simplifying lead to the following equation:

$$\frac{dT_B}{dx} + G_1 T_B = G_2 T_S + G_3 x + G_4 \tag{B12}$$

where

$$G_1 = \frac{h_g l_o}{w_c c_p (1 + \lambda_B)} + \frac{h_{c,I,S} l_S}{w_c c_p}$$

$$G_2 = \frac{h_{c,I,S} l_S}{w_c c_p (1 + \lambda_B)}$$

$$G_3 = \frac{\omega^2}{gJc_p (1 + \lambda_B)}$$

$$G_4 = \frac{\dfrac{h_g l_o}{w_c c_p} + \dfrac{h_{c,I,S} l_S}{w_c c_p} \lambda_B}{1 + \lambda_B} T_{g,e} + G_3 r_h$$

Likewise, equating equations (B9) and (B11), substituting the value of $T_{c,II}$ from equation (2) in equation (B9), and simplifying lead to the desired equation for dT_S/dx:

$$\frac{dT_S}{dx} + G_5 T_S = G_6 T_B + G_7 x + G_8 \tag{B13}$$

where

$$G_5 = -\frac{h_{c,I,S} l_S}{w_c c_p}$$

$$G_6 = (1 + \lambda_B) G_5 - \frac{h_g l_o \lambda_S}{w_c c_p (1 + \lambda_S)}$$

$$G_7 = \frac{\omega^2}{gJc_p}$$

$$G_8 = T_{g,e} \left[\frac{h_g l_o \lambda_S}{(1 + \lambda_S) w_c c_p} + \frac{h_{c,I,S} l_S \lambda_B}{w_c c_p} \right] + G_7 r_h$$

The boundary conditions assigned to equations (B12) and (B13) are

$$\left. \begin{array}{l} \text{at} \quad x = 0 \quad T_{c,I} = T_{c,I_{x=0}} = T_{c,in} \\[10pt] \text{and at} \quad x = b \quad T_{c,I_{x=b}} = T_{c,II_{x=b}} \end{array} \right\} \tag{B14}$$

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Simultaneous first-order linear differential equations with constant coefficients (eqs. (B12) and (B13)) involving two dependent variables (T_B and T_S) and one independent variable x are now available. A solution of these simultaneous differential equations may be obtained by following the procedure outlined on page 312 of reference 18. The solution results in the following two equations:

$$T_B = C_1 Y_1 e^{m_3 \frac{x}{b}} + C_2 Y_2 e^{m_4 \frac{x}{b}} + Y_3 \frac{x}{b} + Y_4 \tag{B15}$$

$$T_S = C_1 e^{m_3 \frac{x}{b}} + C_2 e^{m_4 \frac{x}{b}} - Y_5 \frac{x}{b} + Y_6 \tag{B16}$$

where C_1 and C_2 are integration constants, m_3 and m_4 are real, and Y_1, Y_2, Y_3, Y_4, Y_5, and Y_6 are given by

$$Y_1 = \frac{G_2}{m_1 + G_1}$$

$$Y_2 = \frac{G_2}{m_2 + G_1}$$

$$Y_3 = \frac{\left(G_3 - \frac{G_2 Y_7}{Y_8}\right) b}{G_1}$$

$$Y_4 = \frac{G_1 Y_9 - G_3 + \frac{G_2 Y_7}{Y_8}}{G_1^2}$$

$$Y_5 = \frac{Y_7 b}{Y_8}$$

$$Y_6 = -\left(\frac{Y_8 Y_{10} + Y_{11} Y_7}{Y_8^2}\right)$$

$$Y_7 = G_1 G_7 + G_6 G_3$$

$$Y_8 = G_6 G_2 - G_5 G_1$$

$$Y_9 = G_4 - \frac{G_2}{Y_8}\left(Y_{10} + \frac{Y_{11}Y_7}{Y_8}\right)$$

$$Y_{10} = G_7 + G_1 G_8 + G_6 G_4$$

$$Y_{11} = G_5 + G_1$$

$$m_3 = m_1 b$$

$$m_4 = m_2 b$$

$$m_1 = \frac{-(G_5 + G_1) + \sqrt{(G_5 + G_1)^2 + 4Y_8}}{2}$$

$$m_2 = \frac{-(G_5 + G_1) - \sqrt{(G_5 + G_1)^2 + 4Y_8}}{2}$$

By using equations (1), (2), (B14), and (B15), the coolant temperature distribution equations can be found:

$$T_{c,I} = (1 + \lambda_B)\left(Y_1 C_1 e^{m_3 \frac{x}{b}} + Y_2 C_2 e^{m_4 \frac{x}{b}} + Y_3 \frac{x}{b} + Y_4 - \frac{\lambda_B T_{g,e}}{1 + \lambda_B}\right) \quad \text{(B17)}$$

$$T_{c,II} = (1 + \lambda_S)\left(C_1 e^{m_3 \frac{x}{b}} + C_2 e^{m_4 \frac{x}{b}} - Y_5 \frac{x}{b} + Y_6\right) +$$

$$(1 + \lambda_B)(\lambda_S)\left(\frac{\lambda_B T_{g,e}}{1 + \lambda_B} - Y_1 C_1 e^{m_3 \frac{x}{b}} - Y_2 C_2 e^{m_4 \frac{x}{b}} - Y_3 \frac{x}{b} - Y_4\right) \quad \text{(B18)}$$

The coolant inlet temperature $T_{c,I,in}$ is assumed. Thus, the use of the first boundary conditions of equations (B14) and (B17) yields

$$C_1 = \frac{1}{Y_1}\left[\frac{1}{1 + \lambda_B}\left(T_{c,I,in} + \lambda_B T_{g,e}\right) - Y_2 C_2 - Y_4\right] \quad \text{(B19)}$$

Using the second boundary condition of equation (B14) in (B17) and (B18) and the preceding expression for C_1 yields

$$C_2 = \frac{\frac{e^{m_3}}{Y_1}\left[Y_1(1+\lambda_B) - 1\right]\left(\frac{T_{c,I,in}}{1 + \lambda_B} + \frac{\lambda_B T_{g,e}}{1 + \lambda_B} - Y_4\right) + (1+\lambda_B)(Y_3+Y_4) + Y_5 - Y_6 - \lambda_B T_{g,e}}{\frac{Y_2 e^{m_3}}{Y_1}\left[Y_1(1 + \lambda_B) - 1\right] - e^{m_4}\left[Y_2(1 + \lambda_B) - 1\right]}$$

$$(B20)$$

Equation (B20) allows C_2 to be evaluated when the inlet coolant temperature, the various geometries, and constants are assigned. This, in turn, permits calculation of C_1 from equation (B19), and, hence, T_B, T_S, $T_{c,I}$, and $T_{c,II}$ may be calculated from equations (B15) through (B18), respectively.

Closed-Form Solution Neglecting Heat Transfer Across Blade Insert

Once the heat being transferred across the blade insert is considered negligible, then the blade shell temperature distribution equation derived in reference 5 (considering the same simplifying assumptions made in the present analysis) is applicable for the return-flow blade design described herein. The derivation of reference 5 results in a first-order linear differential equation with constant coefficient. Application of the boundary condition $T_{c,I} = T_{c,in}$ when $x = 0$ results in a closed-form solution for obtaining the spanwise blade shell temperature distribution. The equations which give this blade shell temperature distribution are equations (18) and (20) of reference 5. Equation (18) considers the coolant temperature change due to heat transfer and rotation, and equation (20) considers the change due to heat transfer only. In general, the rotational terms are small and may be neglected. Equation (20), in the notation of this report, is

$$T_B = T_{g,e} - (T_{g,e} - T_{c,in})\frac{1}{1 + \lambda_B} e^{-\left(\frac{1}{1+\lambda_B}\frac{h_g l_{o,B} b}{c_p w_c}\frac{x}{b}\right)}$$

$$(B21)$$

REFERENCES

1. Esgar, J. B., Livingood, J. N. B., and Hickel, R. O.: Research on Application of Cooling to Gas Turbines. Paper No. 56-SA-54, ASME, 1956.

2. Esgar, Jack B., and Ziemer, Robert R.: Review of Status, Methods, and Potentials of Gas-Turbine Air-Cooling. NACA RM E54I23, 1955.

3. Esgar, Jack B., and Ziemer, Robert R.: Effect of Turbine Cooling with
     Compressor Air Bleed on Gas-Turbine Engine Performance. NACA RM
     E54L20, 1955.

4. Esgar, Jack B., and Slone, Henry O.: Gas-Turbine-Engine Performance
     When Heat from Liquid-Cooled Turbines is Rejected Ahead of, Within,
     or Behind Main Compressor. NACA RM E56B09, 1956.

5. Livingood, John N. B., and Brown, W. Byron: Analysis of Spanwise
     Temperature Distribution in Three Types of Air-Cooled Turbine Blades.
     NACA Rep. 994, 1950. (Supersedes NACA RM's E7B11e and E7G30.)

6. Slone, Henry O., Hubbartt, James E., and Arne, Vernon L.: Method of
     Designing Corrugated Surfaces Having Maximum Cooling Effectiveness
     Within Pressure-Drop Limitations for Application to Cooled Turbine
     Blades. NACA RM E54H20, 1954.

7. Livingood, John N. B., and Brown, W. Byron: Analysis of Temperature
     Distribution in Liquid-Cooled Turbine Blades. NACA Rep. 1066, 1952.
     (Supersedes NACA TN 2321.)

8. Hibbard, Robert R.: Evaluation of Liquefied Hydrocarbon Gases as
     Turbojet Fuels. NACA RM E56I21, 1956.

9. Ziemer, Robert R., and Slone, Henry O.: Analytical Procedures for
     Rapid Selection of Coolant Passage Configurations for Air-Cooled
     Turbine Rotor Blades and for Evaluation of Heat-Transfer, Strength,
     and Pressure-Loss Characteristics. NACA RM E52G18, 1952.

10. Hubbartt, James E., Slone, Henry O., and Arne, Vernon L.: Method for
     Rapid Determination of Pressure Change for One-Dimensional Flow
     with Heat-Transfer, Friction, Rotation, and Area Change. NACA TN
     3150, 1954.

11. Slone, Henry O., and Esgar, Jack B.: Gas-to-Blade Heat-Transfer Co-
     efficients and Turbine Heat-Rejection Rates for a Range of One-
     Spool Cooled-Turbine Engine Designs. NACA RM E56A31, 1956.

12. Eckert, E. R. G., and Diaguila, A. J.: Convection Heat Transfer for
     Mixed, Free, and Forced Flow Through Tubes. Trans. ASME, vol. 76,
     no. 4, May 1954, pp. 497-504.

13. Woolley, Harold W., Scott, Russell B., and Brickwedde, F. G.: Com-
     pilation of Thermal Properties of Hydrogen in Its Various Isotopic
     and Ortho-Para Modifications. Jour. Res. Nat. Bur. Standards,
     vol. 41, no. 5, Nov. 1948, pp. 379-475.

14. Anon.: Tables of Thermal Properties of Gases. NBS, July 1949.

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15. Keenan, Joseph H., and Kaye, Joseph: Gas Tables. John Wiley & Sons, Inc., 1948.

16. Pinkel, Benjamin: A Summary of NACA Research on Heat Transfer and Friction for Air Flowing Through Tube with Large Temperature Difference. Trans. ASME, vol. 76, no. 2, Feb. 1954, pp. 305-318.

17. McAdams, William H.: Heat Transmission. Second ed., McGraw-Hill Book Co., Inc., 1942.

18. Sokolnikoff, Ivan S., and Sokolnikoff, Elizabeth S.: Higher Mathematics for Engineers and Physicists. Second ed., McGraw-Hill Book Co., Inc., 1941.

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TABLE I. - COOLANT REQUIREMENTS FOR RECIRCULATING-TYPE

HOLLOW-INSERT TURBINE BLADE

[Turbine-inlet temperature, 3000° R; flight Mach
number, 2.5; blade root stress, 50,000 psi;
stress-ratio factor, 1.5; spacing between blade
shell and insert, 0.050 in.]

(a) Coolant, hydrogen

Gas-to-blade heat-transfer coefficient, h_g, Btu/(sec)(sq ft)($^{\circ}$F)	Coolant inlet temperature, $T_{c,in}$, $^{\circ}$R	Required coolant flow per blade, w_c, lb/sec	Coolant inlet static pressure, $P_{c,in}$, lb/sq ft	Coolant static pressure at blade tip, $P_{c,I,b}$, lb/sq ft	Coolant outlet static pressure, $P_{c,out}$, lb/sq ft	Coolant flow regime
0.0580	250	0.0244	3000	2738	2,943	Transitional
			6000	6261	7,624	
			9000	9572	11,790	
	500	0.0171	3000	2657	2,945	Transitional
			6000	6051	7,472	
			9000	9248	11,540	
	750	0.0165	3000	2416	2,387	Transitional
			6000	5910	7,289	
			9000	9094	11,380	
	1000	0.0194	3000	(a)	(a)	Transitional
			6000	5668	6,862	
			9000	8914	11,110	
0.0213	250	0.0017	3000	3090	3,911	Laminar
			6000	6224	7,891	
			9000	9387	11,900	
	500	0.0063	3000	3050	3,774	Transitional
			6000	6150	7,610	
			9000	9237	11,410	
	750	0.0061	3000	3026	3,822	Transitional
			6000	6117	7,748	
			9000	9195	11,640	
	1000	0.0071	3000	2994	3,776	Transitional
			6000	6090	7,719	
			9000	9165	11,620	

a No solution.

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TABLE I. - Continued.   COOLANT REQUIREMENTS FOR RECIRCULATING-
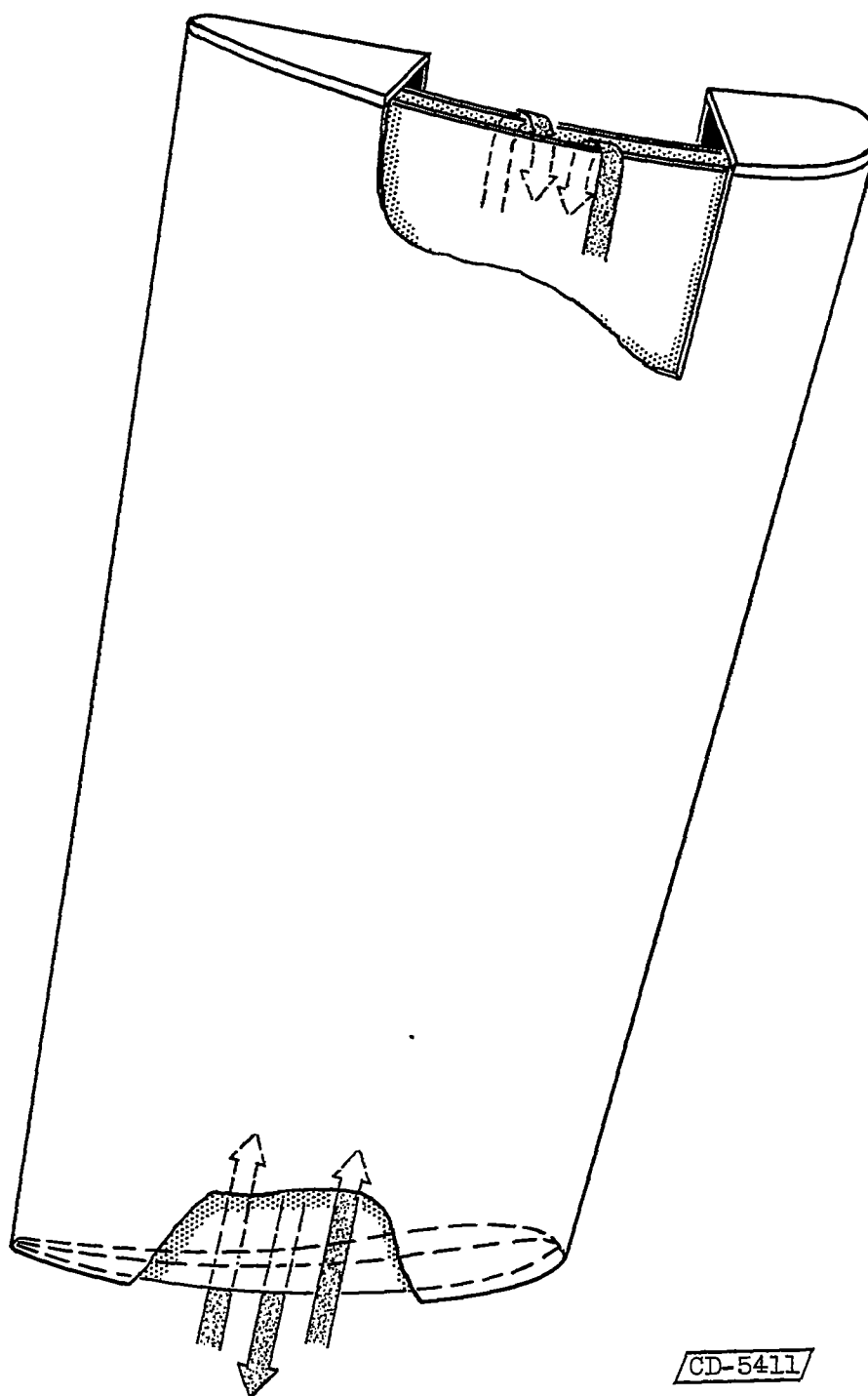
TYPE HOLLOW-INSERT TURBINE BLADE

[Turbine-inlet temperature, 3000° R; flight Mach number,
2.5; blade root stress, 50,000 psi; stress-ratio
factor, 1.5; spacing between blade shell and insert,
0.050 in.]

(b) Coolant, helium

Gas-to-blade heat-transfer coefficient, h_g, Btu/(sec)(sq ft)(°F)	Coolant inlet temperature, $T_{c,in}$, °R	Required coolant flow per blade, w_c, lb/sec	Coolant inlet static pressure, $P_{c,in}$, lb/sq ft	Coolant static pressure at blade tip, $P_{c,I,b}$, lb/sq ft	Coolant outlet static pressure, $P_{c,out}$, lb/sq ft	Coolant flow regime
0.0580	250	0.0675	3000	(a)	(a)	Transitional
			6000	6077	6,390	
			9000	9897	11,280	
	500	0.0476	3000	(a)	(a)	Transitional
			6000	5663	6,190	
			9000	9248	10,890	
	750	0.0463	3000	(a)	(a)	Transitional
			6000	(a)	(a)	
			9000	8553	10,011	
	1000	0.0534	3000	(a)	(a)	Transitional
			6000	(a)	(a)	
			9000	8185	8,376	
0.0213	250	0.0248	3000	3100	3,850	Transitional
			6000	6402	7,963	
			9000	9690	12,050	
	500	0.0175	3000	3002	3,463	Transitional
			6000	6388	7,701	
			9000	9681	11,744	
	750	0.0171	3000	2844	3,236	Transitional
			6000	6204	7,543	
			9000	9435	11,569	
	1000	0.0201	3000	2474	2,439	Transitional
			6000	6008	7,275	
			9000	9245	11,360	

[a]No solution.

TABLE I. - Concluded.   COOLANT REQUIREMENTS FOR RECIRCULATING-

TYPE HOLLOW-INSERT TURBINE BLADE

[Turbine-inlet temperature, 3000° R; flight Mach number,
2.5; blade root stress, 50,000 psi; stress-ratio
factor, 1.5; spacing between blade shell and insert,
0.050 in.]

(c) Coolant, air

Gas-to-blade heat-transfer coefficient, h_g, Btu/(sec)(sq ft)(°F)	Coolant inlet temperature, $T_{c,in}$, °R	Required coolant flow per blade, w_c, lb/sec	Coolant inlet static pressure, $P_{c,in}$, lb/sq ft	Coolant static pressure at blade tip, $P_{c,I,b}$, lb/sq ft	Coolant outlet static pressure, $P_{c,out}$, lb/sq ft	Coolant flow regime
0.0580	250	0.3629	3000	(a)	(a)	Turbulent
			6000	(a)	(a)	
			9000	23,022	11,885	
	500	0.2551	3000	(a)	(a)	Turbulent
			6000	(a)	(a)	
			9000	14,720	10,085	
	750	0.2528	3000	(a)	(a)	Turbulent
			6000	(a)	(a)	
			9000	11,800	7,094	
	1000	0.3129	3000	(a)	(a)	Turbulent
			6000	(a)	(a)	
			9000	(a)	(a)	
0.0213	250	0.1165	3000	7,336	7,247	Turbulent
			6000	14,895	15,670	
			9000	22,396	23,790	
	500	0.0735	3000	4,718	3,450	Transitional
			6000	9,970	8,223	
			9000	15,087	12,630	
	750	0.0730	3000	3,819	2,589	Transitional
			6000	8,572	7,633	
			9000	13,068	11,890	
	1000	0.0901	3000	(a)	(a)	Transitional
			6000	7,707	6,905	
			9000	11,987	11,270	

[a]No solution.
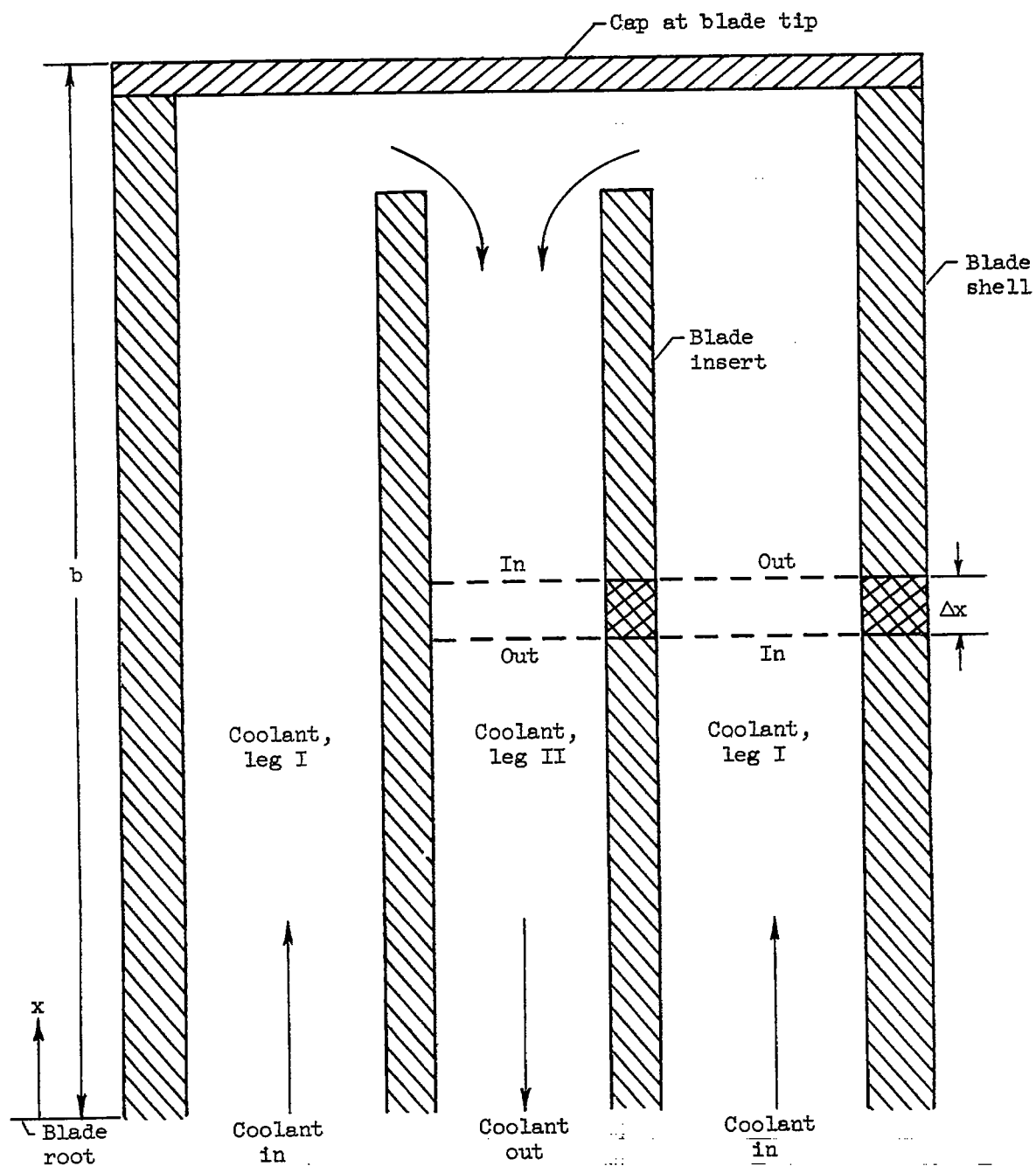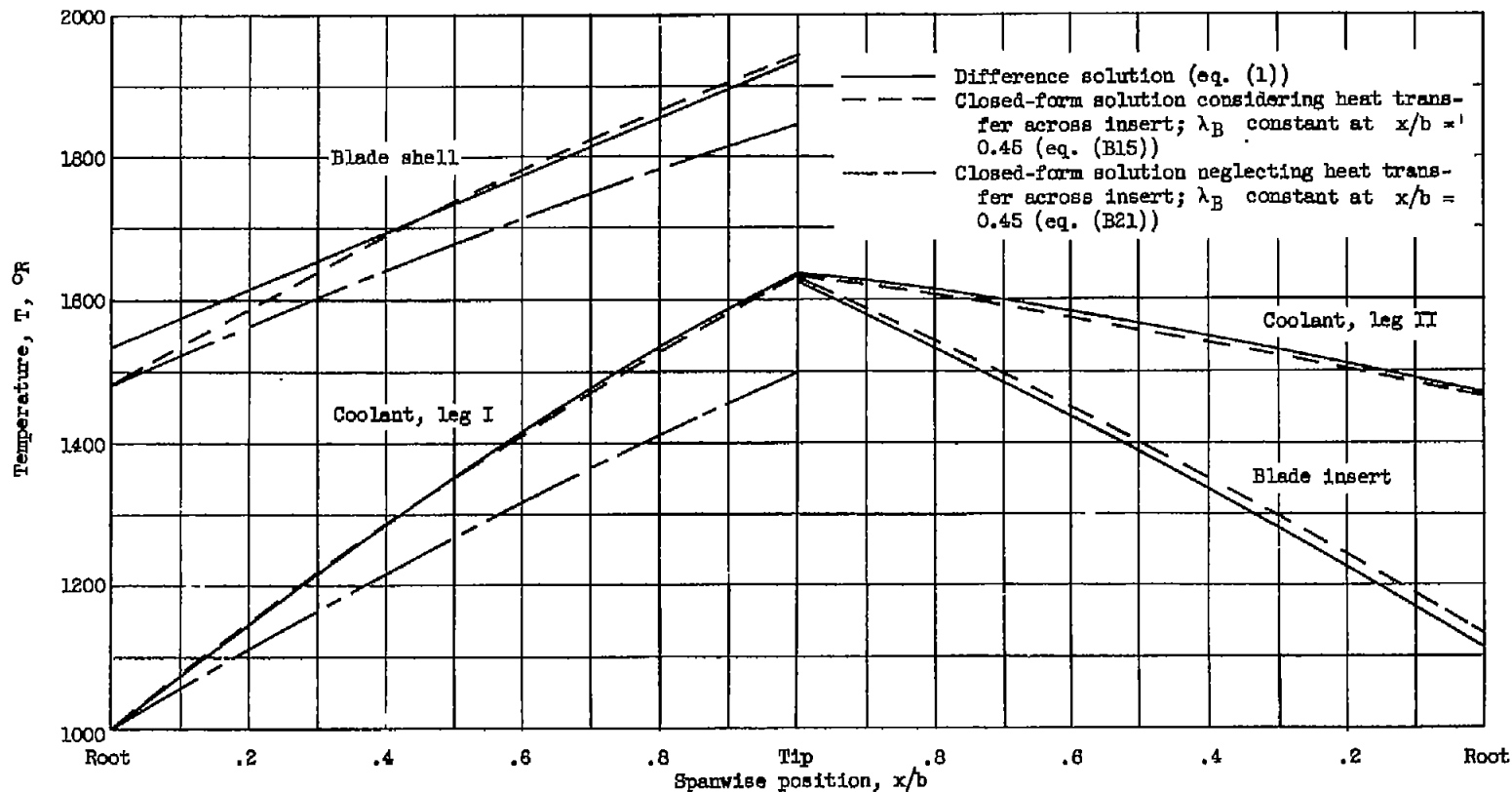
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Figure 1. - Sketch of return-flow blade.

Figure 2. - Schematic diagram of recirculating-type hollow turbine blade with insert.

(a) Coolant, air; coolant inlet temperature, 1000° R; coolant flow per blade, 0.1235 pound per second.

Figure 3. - Comparison of difference-solution equations and closed-form-solution equations for spanwise temperature distributions. Spacing between blade shell and insert, 0.020 inch; gas-to-blade heat-transfer coefficient, 0.0580 Btu/(sec)(sq ft)(°F); stress-ratio factor, 1.5.
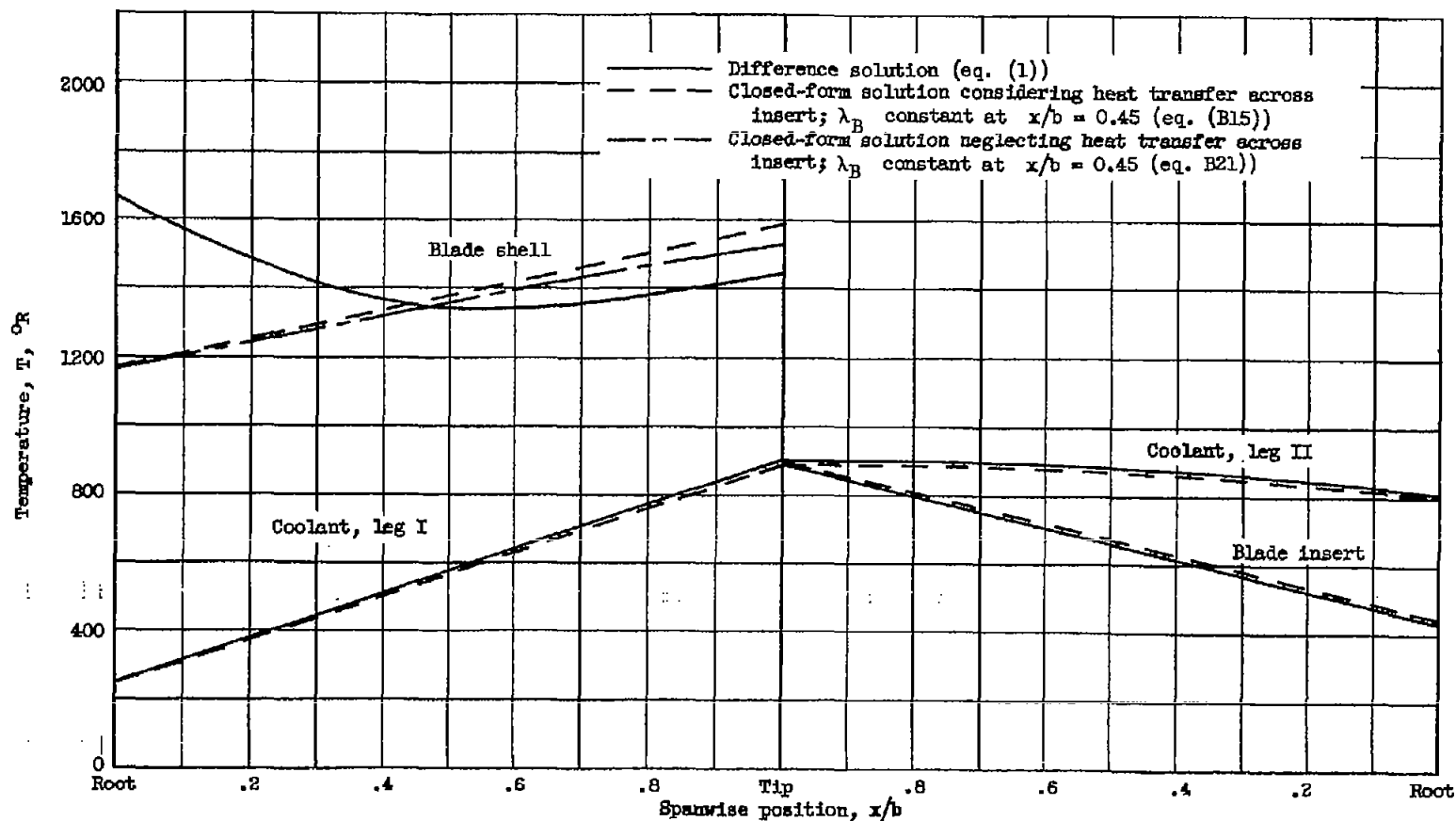
(b) Coolant, hydrogen; coolant inlet temperature, 250° R; coolant flow per blade, 0.0099 pound per second.

Figure 3. - Concluded. Comparison of difference-solution equations and closed-form-solution equations for spanwise temperature distributions. Spacing between blade shell and insert, 0.020 inch; gas-to-blade heat-transfer coefficient, 0.0580 Btu/(sec)(sq ft)(°F); stress-ratio factor, 1.5.

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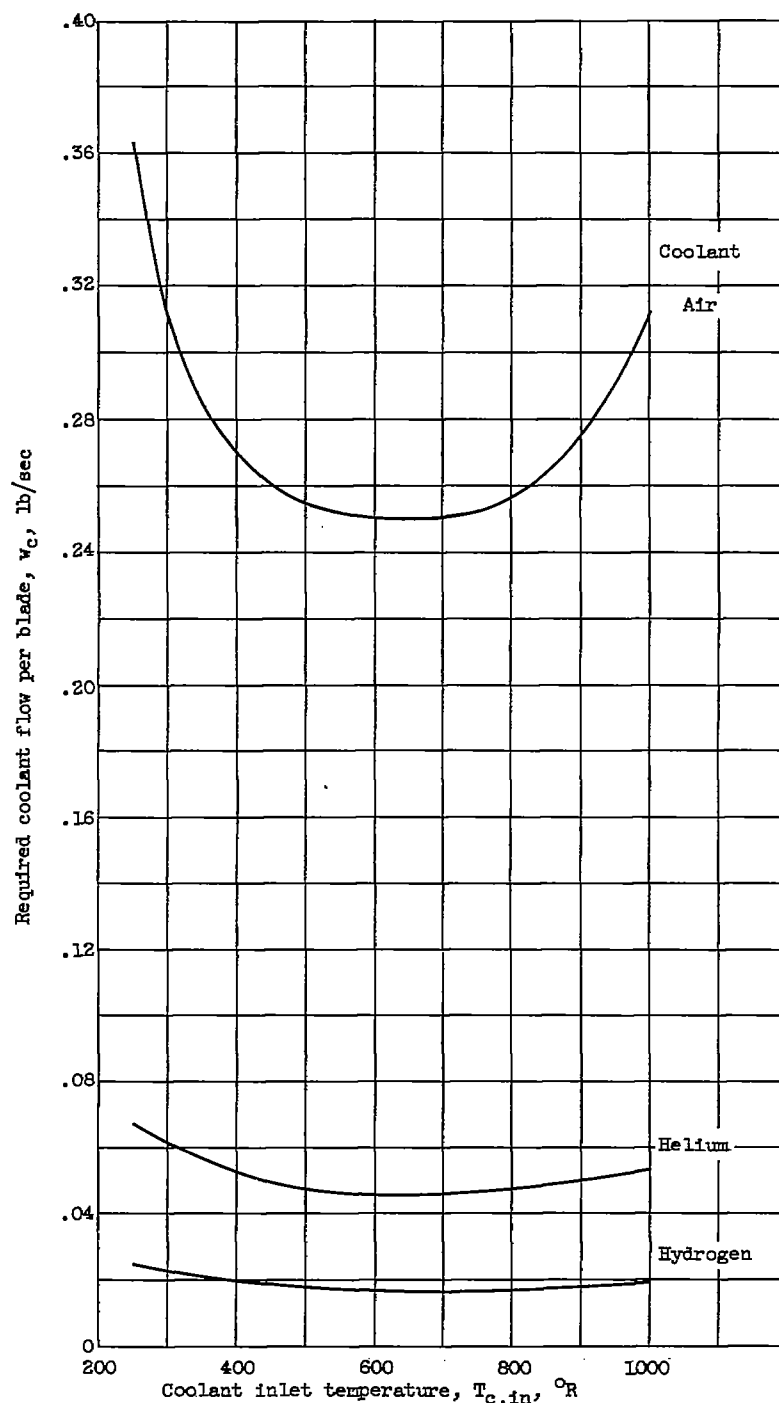
Figure 4. - Variation in required coolant flow
with coolant inlet temperature. Spacing be-
tween blade shell and insert, 0.050 in.; gas-
to-blade heat-transfer coefficient, 0.0580
Btu/(sec)(sq ft)(^{O}F); stress-ratio factor,
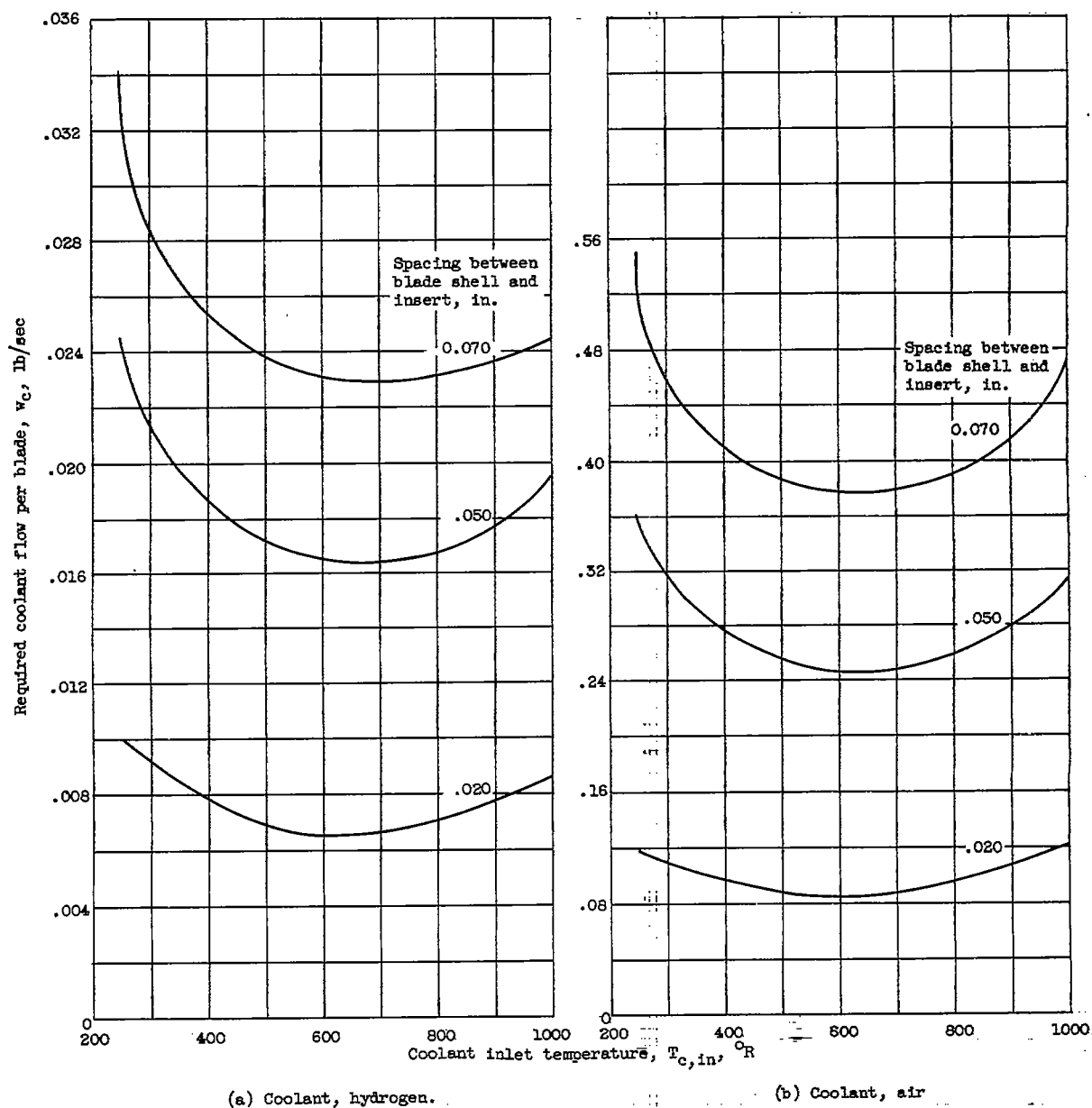1.5.

(a) Coolant, hydrogen.

(b) Coolant, air

Figure 5. - Variation in required coolant flow with coolant inlet temperature and three values of coolant geometry spacing. Gas-to-blade heat-transfer coefficient, 0.0580 Btu/(sec)(sq ft)(°F); stress-ratio factor, 1.5.
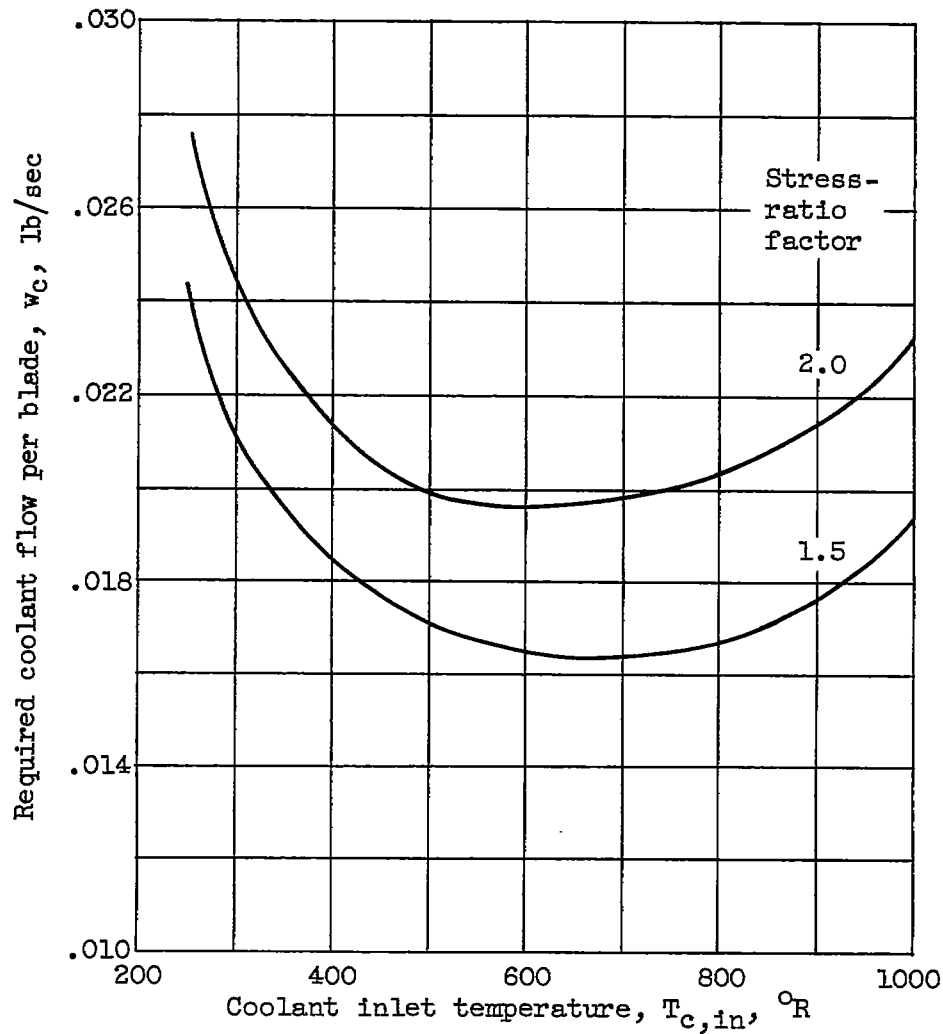
Figure 6. - Variation in required coolant flow with
coolant inlet temperature for two values of stress-
ratio factor. Coolant, hydrogen; spacing between
blade shell and insert, 0.050 inch; gas-to-blade
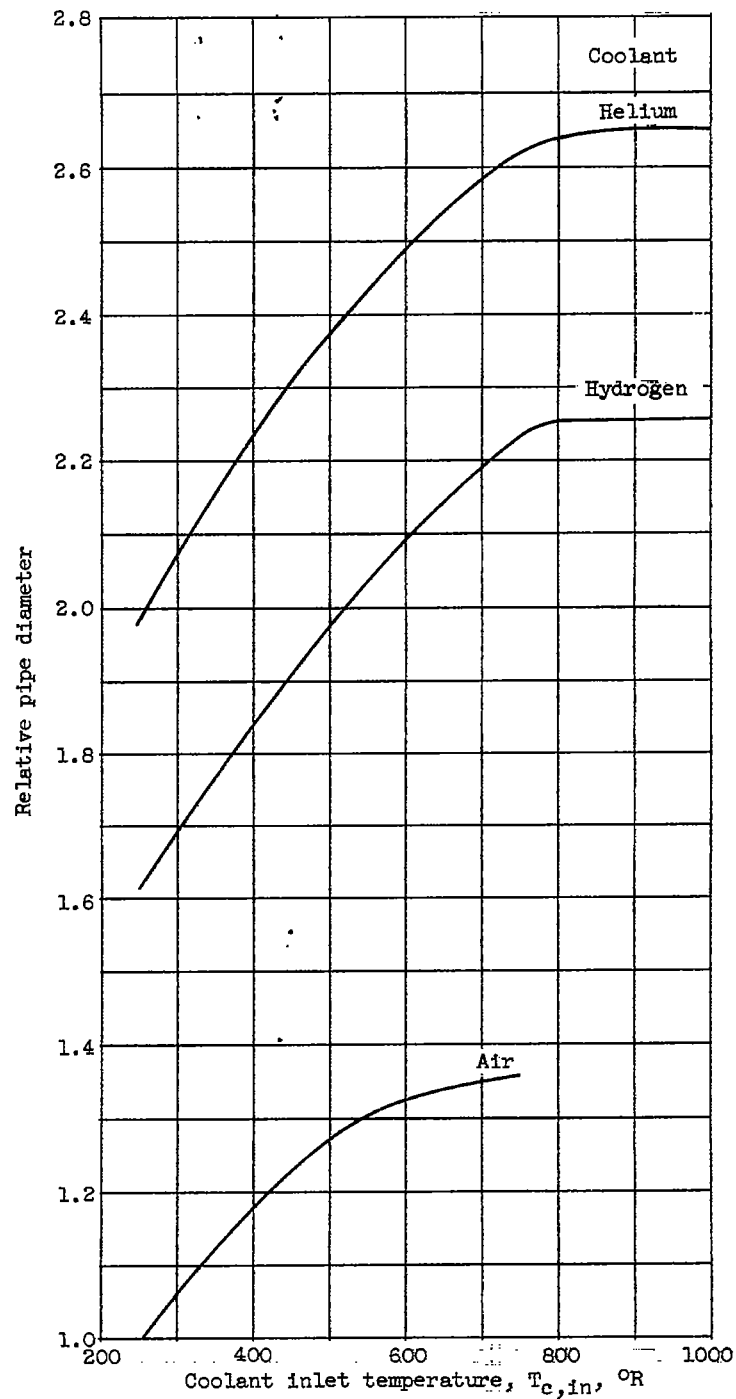heat-transfer coefficient, 0.0580 Btu/(sec)(sq ft)($^{\circ}$F).

Figure 7. - Variation of relative pipe diameter with coolant inlet temperature for conditions of constant pipe pressure loss. Gas-to-blade heat-transfer coefficient, 0.058 Btu/(sec) (sq ft)($^{\circ}$F).